**Name:** *Solutions* **Student Number:** ________________

**Mathematics 1110H,** TRENT UNIVERSITY, Fall 2025.

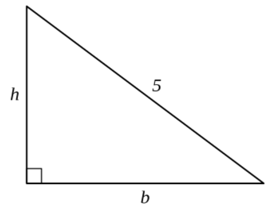
**Test #4** for Section F01 and CAT. *Friday, 14 November.* Time: 20 minutes.

### Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do one (1) of questions 1 or 2.** (Question **2** is on page 2.)

**1.** What is the maximum area of a right triangle with a hypotenuse of length 5 $m$? *[10]*

SOLUTIONS. A triangle with base $b$ and height $h$ has area $A = \frac{bh}{2}$; in the case of a right triangle, we can take each of the base and height to be one of the short sides of the triangle.




Since the triangle is right, the Pythagorean Theorem tells us that $b^2 + h^2 = 5^2 = 25$, so $h = \sqrt{25 - b^2}$. Note that we must take the positive square root because both $b$ and $h$ must be $\geq 0$. Also, both must be less than or equal to the length of the hypotenuse, which is 5, so $0 \leq b \leq 5$ and $0 \leq h \leq 5$.

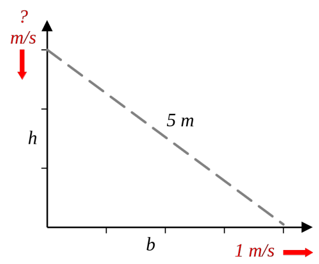
Putting the various pieces above together, we need to find the maximum of $A = \frac{bh}{2} = \frac{h\sqrt{25 - h^2}}{2}$ on the interval $[0, 5]$. At the endpoints we have $A(0) = \frac{0\sqrt{25 - 0^2}}{2} \frac{0 \cdot 5}{2} = 0$ and $A(5) = \frac{5\sqrt{25-5^2}}{2} = \frac{5 \cdot 0}{2} = 0$. We next check for critical points in the interval.

$$
\begin{aligned}
A'(h) &= \frac{d}{dh}\left(\frac{h\sqrt{25 - h^2}}{2}\right) = \frac{\left[\frac{dh}{dh}\right]\sqrt{25 - h^2} + h\left[\frac{d}{dh}\sqrt{25 - h^2}\right]}{2} \\
&= \frac{[1]\sqrt{25 - h^2} + h\left[\frac{1}{2\sqrt{25-h^2}} \cdot \frac{d}{dh}\left(25 - h^2\right)\right]}{2} \\
&= \frac{\sqrt{25 - h^2} + \frac{h}{2\sqrt{25-h^2}} \cdot (-2h)}{2} = \frac{\sqrt{25 - h^2} - \frac{h^2}{\sqrt{25-h^2}}}{2} \cdot \frac{\sqrt{25 - h^2}}{\sqrt{25 - h^2}} \\
&= \frac{25 - h^2 - h^2}{2\sqrt{25 - h^2}} = \frac{25 - 2h^2}{2\sqrt{25 - h^2}} = 0 \iff 25 - 2h^2 = 0 \iff h = \pm\frac{5}{\sqrt{2}}
\end{aligned}
$$

The critical point $h = 5/\sqrt{2} \approx 3.5355$ is in the interval $[0, 5]$, but its negative is not. Since the area at the only critical point $A\left(\frac{5}{\sqrt{2}}\right) = \frac{\frac{5}{\sqrt{2}}\sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2}}{2} = \frac{\frac{5}{\sqrt{2}}\sqrt{\frac{25}{2}}}{2} = \frac{5 \cdot 5}{2\sqrt{2}\sqrt{2}} = \frac{25}{4} = 6.25$ is greater than the areas at each endpoint, the maximum area of such a triangle is $6.25\ m^2$. ■

**2.** Initially, a 5 $m$ long ladder is put up flush against a vertical wall. Its base then begins to slide away from the wall along the horizontal floor at a constant rate of 1 $m/s$, with the top of the ladder maintaining contact with the wall throughout. How is the top of the ladder moving at the instant that it is 3 $m$ above the floor? *[10]*

SOLUTION. At any given instant, the wall, the floor, and the ladder form a right triangle.




If at any given instant the top of the ladder is $h$ $m$ above the ground and the base of the ladder is $b$ $m$ from the wall, then we have $h^2 + b^2 = 5^2 = 25$ by the Pythagorean Theorem and we are given that $\frac{db}{dt} = 1\ m/s$. We are asked to find $\frac{dh}{dt}$ at the instant that $h = 3\ m$.

When $h = 3$, $3^2 + b^2 = 25$, so $b^2 = 25 - 9 = 16$, so $b = 4$. (We take the positive root of 16 because $b$ is a length.) Also, at any given instant we have:

$$\begin{aligned} h^2 + b^2 = 25 &\implies \frac{d}{dt}h^2 + \frac{d}{dt}b^2 = \frac{d}{dt}25 \\ &\implies \left[\frac{d}{dh}h^2\right]\frac{dh}{dt} + \left[\frac{d}{db}b^2\right]\frac{db}{dt} = 0 \\ &\implies 2h\frac{dh}{dt} + 2b\frac{db}{dt} = 0 \\ &\implies \frac{dh}{dt} = -\frac{2b}{2h} \cdot \frac{db}{dt} = -\frac{b}{h} \cdot \frac{db}{dt} \end{aligned}$$

At the instant that $h = 3\ m$, we have $b = 4\ m$ and $\frac{db}{dt} = 1\ m/s$. Thus, at this instant we have:

$$\left.\frac{dh}{dt}\right|_{h=3} = -\frac{4}{3} \cdot 1 = -\frac{4}{3}\ m/s \approx -1.33333\ m/s$$

Thus the top of the ladder is coming down at a rate of $\frac{4}{3}$ $m/s$ at the instant that it is 3 $m$ above the floor. ■