Name: *Solutions* **Student Number:** ________________

Mathematics 1110H, TRENT UNIVERSITY, Fall 2025. **The Dread Halloween Test**
a.k.a. **Test #3** for Sections F02 and F04. *Friday, 31 October.* Time: 40 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do question 1.** There is no other ...

1. Find the domain, as well as any and all intercepts, vertical and horizontal asymptotes, intervals of increase and decrease, local maximum and minimum points, intervals of concavity, and inflection points of $f(x) = x + \dfrac{1}{x}$. *[5]*

SOLUTION. *i. Domain.* Thanks to the x in the denominator of $\dfrac{1}{x}$, the domain of $f(x) = x + \dfrac{1}{x}$ is all $x \neq 0$, *i.e.* it is $\{\, x \in \mathbb{R} \mid x \neq 0 \,\}$. Note that $f(x)$ is continuous and differentiable wherever it is defined.

ii. Intercepts. 0 is not in the domain of $f(x)$, so it has no y-intercept. On the other hand $f(x) = x + \dfrac{1}{x} = 0$ exactly when $x^2 + 1 = 0$, which is impossible because $x^2 + 1 \geq 1$ for all x, so $f(x)$ also has no x-intercepts.

iii. Vertical asymptotes. Since $f(x)$ is continuous everywhere except at $x = 0$, we check for possible vertical asymptotes there:

$$\lim_{x \to 0^-} \left(x + \frac{1}{x} \right) = 0^- + (-\infty) = -\infty$$
$$\lim_{x \to 0^+} \left(x + \frac{1}{x} \right) = 0^+ + (+\infty) = +\infty$$

Thus $f(x)$ has a vertical asymptotes at $x = 0$, going down coming from the left and going up coming from the right.

iv. Horizontal asymptotes. We take the limits in both directions and see what happens:

$$\lim_{x \to -\infty} \left(x + \frac{1}{x} \right) = -\infty + 0^- = -\infty$$
$$\lim_{x \to 0^+} \left(x + \frac{1}{x} \right) = +\infty + 0^+ = +\infty$$

Thus $f(x)$ does not have any horizontal asymptotes.

v. Intervals of increase and decrease, and maxima and minima. We first compute the derivative of $f(x)$,

$$f'(x) = \frac{d}{dx}\left(x + \frac{1}{x} \right) = \frac{d}{dx}\left(x + x^{-1} \right) = 1 - x^{-2} = 1 - \frac{1}{x^2},$$

and then use it. Observe that

$$f'(x) = 1 - \frac{1}{x^2} \begin{matrix} < 0 \\ = 0 \\ > 0 \end{matrix} \iff \begin{matrix} x^2 < 1 \\ x^2 = 1 \\ x^2 > 1 \end{matrix} \iff \begin{matrix} -1 < x < 1 \text{ and } x \neq 0 \\ x = \pm 1 \\ x < -1 \text{ or } x > 1 \end{matrix},$$

from which it follows that $f(x) = x + \dfrac{1}{x}$ is decreasing when $-1 < x < 1$ (and $x \neq 0$), has critical points at $x = \pm 1$, and is increasing when $x < -1$ and when $x > 1$. It follows in turn that $x = -1$ is a (local) maximum and $x = 1$ is a (local) minimum. We summarize this in the usual table:

x	$(-\infty, -1)$	-1	$(-1, 0)$	0	$(0, 1)$	1	$(1, \infty)$
$f'(x)$	$+$	0	$-$	undef	$-$	0	$+$
$f(x)$	$\uparrow$	max	$\downarrow$	undef	$\downarrow$	min	$\uparrow$

vi. Intervals of concavity and points of inflection. We first compute the second derivative of $f(x)$,

$$f''(x) = \frac{d}{dx} f'(x) = \frac{d}{dx}\left(1 - \frac{1}{x^2}\right) = \frac{d}{dx}\left(1 - x^{-2}\right) = 0 - \left(-2x^{-3}\right) = \frac{2}{x^3},$$

and then use it. Note that $f''(x)$ is never 0, is undefined at $x = 0$, and is negative when $x < 0$ and positive when $x > 0$. Thus $f(x) = x + \dfrac{1}{x}$ is concave down on $(-\infty, 0)$, undefined at $x = 0$, and concave up on $(0, \infty)$, and so has no inflection points. We summarize this in another table:

x	$(-\infty, 0)$	0	$(0, \infty)$
$f''(x)$	$-$	undef	$+$
$f(x)$	$\frown$	undef	$\smile$

vii. Graph. It wasn't actually asked for, so we feel no remorse about having SageMath plot it.

[1]:
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plot( x + 1/x, -5, 5, ymin=-5, ymax=5 )
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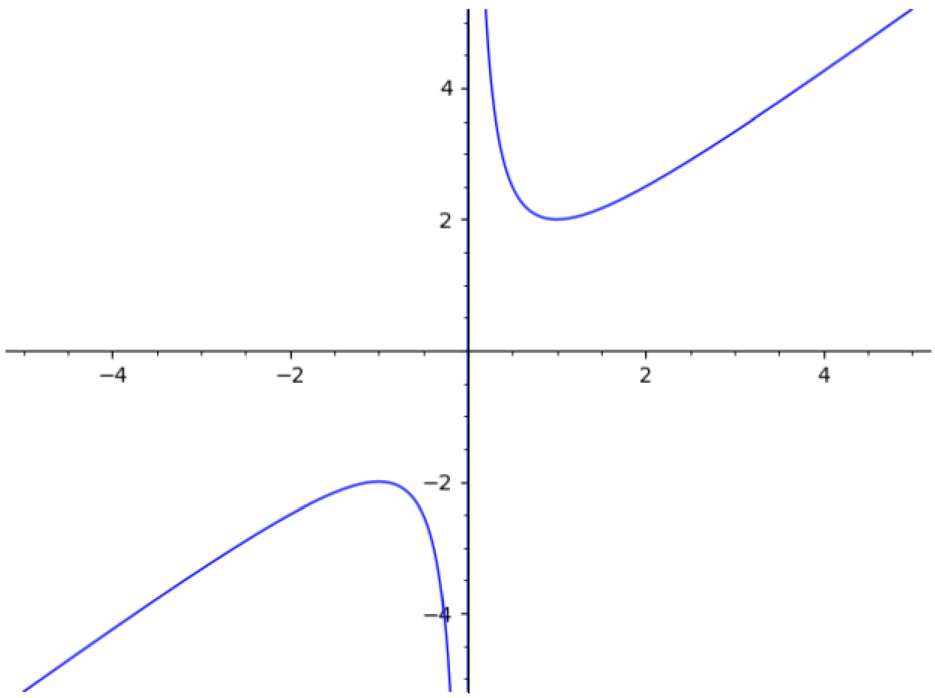
[1]:



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