Name: *Solutions* **Student Number:** ______________

Mathematics 1110H, Trent University, Fall 2025

Test #2 for Sections F03 and F05. *Friday, 10 October.* Time: 20 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do both (2) of questions 1 and 2.**

1. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = x^2 - 6x + 5$. *[5]*

Solution. By definition, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. We plug $f(x) = x^2 - 6x + 5$ into this definition and see what happens:

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\left[(x+h)^2 - 6(x+h) + 5\right] - \left[x^2 - 6x + 5\right]}{h} \\
&= \lim_{h \to 0} \frac{\left[x^2 + 2hx + h^2 - 6x - 6h + 5\right] - \left[x^2 - 6x + 5\right]}{h} \\
&= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 6x - 6h + 5 - x^2 + 6x - 5}{h} \\
&= \lim_{h \to 0} \frac{2hx + h^2 - 6h}{h} \\
&= \lim_{h \to 0} (2x + h - 6) \\
&= 2x + 0 - 6 \\
&= 2x - 6
\end{aligned}$$

Thus, by the limit definition of the derivative, $f'(x) = 2x - 6$ for $f(x) = x^2 - 6x + 5$. ∎

Question **2** *is on the back.*

Question **1** *is on the front.*

2. Use the derivative rules to compute $f'(x)$ if $f(x) = \dfrac{e^{5x}+e^{3x}}{2e^{4x}+2e^{2x}}$. Simplify your answer as much as you reasonably can. *[5]*

SOLUTION 1. *Simplify first.* We can simplify $f(x)$ quite a bit using algebra:

$$f(x) = \frac{e^{5x}+e^{3x}}{2e^{4x}+2e^{2x}} = \frac{e^{3x}e^{2x}+e^{3x}}{2\left(e^{2x}e^{2x}+e^{2x}\right)} = \frac{e^{3x}\left(e^{2x}+1\right)}{2e^{2x}\left(e^{2x}+1\right)} = \frac{e^{3x}}{2e^{2x}} = \frac{e^{2x}e^{x}}{2e^{2x}} = \frac{e^{x}}{2}$$

Thus $f'(x) = \dfrac{d}{dx}\left(\dfrac{e^x}{2}\right) = \dfrac{e^x}{2}$. $\square$

SOLUTION 2. *Use the Quotient and Chain Rules and hope for the best.* Here we go:

$$\begin{aligned}
f'(x) &= \frac{d}{dx}\left(\frac{e^{5x}+e^{3x}}{2e^{4x}+2e^{2x}}\right)\\
&= \frac{\left[\frac{d}{dx}\left(e^{5x}+e^{3x}\right)\right]\left(2e^{4x}+2e^{2x}\right)-\left(e^{5x}+e^{3x}\right)\left[\frac{d}{dx}\left(2e^{4x}+2e^{2x}\right)\right]}{\left(2e^{4x}+2e^{2x}\right)^2}\\
&= \frac{\left[e^{5x}\frac{d}{dx}(5x)+e^{3x}\frac{d}{dx}(3x)\right]\left(2e^{4x}+2e^{2x}\right)-\left(e^{5x}+e^{3x}\right)\left[2e^{4x}\frac{d}{dx}(4x)+2e^{2x}\frac{d}{dx}(2x)\right]}{\left(2e^{4x}+2e^{2x}\right)^2}\\
&= \frac{\left[5e^{5x}+3e^{3x}\right]\left(2e^{4x}+2e^{2x}\right)-\left(e^{5x}+e^{3x}\right)\left[8e^{4x}+4e^{2x}\right]}{\left(2e^{4x}+2e^{2x}\right)^2}\\
&= \frac{\left(10e^{9x}+10e^{7x}+6e^{7x}+6e^{5x}\right)-\left(8e^{9x}+4e^{7x}+8e^{7x}+4e^{5x}\right)}{\left(2e^{4x}+2e^{2x}\right)^2}\\
&= \frac{2e^{9x}+4e^{7x}+2e^{5x}}{\left(2e^{2x}\left[e^{2x}+1\right]\right)^2} = \frac{2e^{5x}\left(e^{4x}+2e^{2x}+1\right)}{\left(2e^{2x}\right)^2\left[e^{2x}+1\right]^2} = \frac{2e^{5x}\left(e^{2x}+1\right)^2}{4e^{4x}\left[e^{2x}+1\right]^2} = \frac{2e^{5x}}{4e^{4x}} = \frac{e^x}{2}
\end{aligned}$$

Whew! Thus $f'(x) = \dfrac{d}{dx}\left(\dfrac{e^x}{2}\right) = \dfrac{e^x}{2}$. ■