Name: *Solutions* **Student Number:** ____________

Mathematics 1110H, TRENT UNIVERSITY, Fall 2025

Test #2 for Sections F02 and F04. *Friday, 10 October.* Time: 20 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do both (2) of questions 1 and 2.**

1. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = x^2 + 2x + 1$. *[5]*

SOLUTION. By definition, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. We plug $f(x) = 2x^2 - x$ into this definition and see what happens:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{\left[(x+h)^2 + 2(x+h) + 1\right] - \left[x^2 + 2x + 1\right]}{h} \\ &= \lim_{h \to 0} \frac{\left[x^2 + 2hx + h^2 + 2x + 2h + 1\right] - \left[x^2 + 2x + 1\right]}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + 2x + 2h + 1 - x^2 - 2x - 1}{h} \\ &= \lim_{h \to 0} \frac{2hx + h^2 + 2h}{h} \\ &= \lim_{h \to 0} (2x + h + 2) \\ &= 2x + 0 + 2 \\ &= 2x + 2 \end{aligned}$$

Thus, by the limit definition of the derivative, $f'(x) = 2x + 2$ if $f(x) = x^2 + 2x + 1$. ■

*Question **2** is on the back.*

Question **1** *is on the front.*

2. Use the derivative rules to compute $f'(x)$ if $f(x) = \sec^2(\arctan(x))$. Simplify your answer as much as you reasonably can. *[5]*

SOLUTION 1. *Simplify first.* We will use the trigonometric identity $1 + \tan^2(t) = \sec^2(t)$ to simplify $f(x)$ first:

$$f(x) = \sec^2(\arctan(x)) = 1 + \tan^2(\arctan(x)) = 1 + [\tan(\arctan(x))]^2 = 1 + x^2$$

Thus, with the help of the Power Rule, $f'(x) = \frac{d}{dx}\left(1 + x^2\right) = 2x$. □

SOLUTION 2. *Use the Power and Chain Rules and hope for the best.*

$$\begin{aligned}
f'(x) &= \frac{d}{dx}\sec^2(\arctan(x)) \\
&= 2\sec(\arctan(x)) \cdot \frac{d}{dx}\sec(\arctan x) \\
&= 2\sec(\arctan(x)) \cdot \sec(\arctan x)\tan(\arctan x) \cdot \frac{d}{dx}\arctan(x) \\
&= 2\sec^2(\arctan x) \cdot x \cdot \frac{1}{1+x^2} \\
&= 2x\left[1 + \tan^2(\arctan(x))\right] \cdot \frac{1}{1+x^2} \\
&= 2x\left[1 + (\tan(\arctan(x)))^2\right] \cdot \frac{1}{1+x^2} \\
&= 2x\left[1 + x^2\right] \cdot \frac{1}{1+x^2} \\
&= 2x
\end{aligned}$$

Thus $f'(x) = 2x$. ■