Name: Solutions **Student Number:** ____________

Mathematics 1110H, TRENT UNIVERSITY, Fall 2025

Test #1 for Sections F03 and F05. *Friday, 26 September.* Time: 20 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do any two (2) of questions 1–3.** *If you do all three and don't cross one out, only the first two encountered by the graders will be marked.*

1. Find the x-intercepts and the location of the tip of the parabola $y = x^2 - 6x - 7$. *[5]*

SOLUTION I. *Intercepts first.* If you spot that $y = x^2 - 6x - 7 = (x+1)(x-7)$, it is easy to see that the x-intercepts of the parabola, the x values that make $y = 0$, are $x = -1$ and $x = 7$. If you don't spot the factorization, you can use the quadratic formula to find the x-intercepts:

$$y = x^2 - 6x - 7 = 0 \implies x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot (-7)}}{2 \cdot 1} = \frac{6 \pm \sqrt{36 - (-28)}}{2}$$
$$= \frac{6 \pm \sqrt{94}}{2} = \frac{6 \pm 8}{2} = \frac{-2}{2} = -1 \text{ or } = \frac{14}{2} = 7$$

Either way, the x-coordinate of the tip of the parabola is halfway between the intercepts, *i.e.* $x = \frac{-1+7}{2} = \frac{6}{2} = 3$. We plug this into the parabola's equation to get the y-coordinate of the tip: $y = 3^2 - 6 \cdot 3 - 7 = 9 - 18 - 7 = -16$ Thus the tip of the parabola is at the point $(3, -16)$. □

SOLUTION II. *Tip first.* We shall find the coordinates of the tip of the parabola by completing the square in the equation defining the parabola:

$$y = x^2 - 6x - 7 = \left(x + \frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 - 7 = (x-3)^2 - (-3)^2 - 7$$
$$= (x-3)^2 - 9 - 7 = (x-3)^2 - 16$$

Thus the tip of the parabola is at the point $(3, -16)$: the x-coordinate 3 makes the squared part 0, leaving -16 as the y-coordinate.

For the x-intercepts, we could factor the quadratic or apply the quadratic formula as in the solution above. Alternatively, we could observe that in the completed square form, $y = 0$ exactly when $(x-3)^2 - 16 = 0$, *i.e.* exactly when $(x-3)^2 = 16$. This means that $x - 3 = \pm\sqrt{16} = \pm 4$, so $x = -4 + 3 = -1$ or $x = 4 + 3 = 7$. Thus the x-intercepts of the parabola are $x = -1$ and $x = 7$. ■

2. Use the ε–δ definition of limits to verify that $\lim_{x \to 3} (3x+3) = 12$. *[5]*

SOLUTION. We need to check that for any given $\varepsilon > 0$, there is a $\delta > 0$, such that if $|x-3| < \delta$, then $|(3x+3)-12| < \varepsilon$. As usual we try to reverse-engineer the necessary δ from the desired conclusion:

$$\begin{aligned} |(3x+3)-12| < \varepsilon \iff |3x-9| < \varepsilon \iff |3(x-3)| < \varepsilon \\ \iff 3|x-3| < \varepsilon \iff |x-3| < \frac{\varepsilon}{3} \end{aligned}$$

If we set $\delta = \frac{\varepsilon}{3}$, then if $|x-3| < \delta$, we are guaranteed to have $|(3x+3)-12| < \varepsilon$, because every step in the reverse-engineering process above is fully reversible.

Since we can find a $\delta > 0$ for any $\varepsilon > 0$, $\lim_{x \to 3} (3x+3) = 12$ satisfies the ε–δ definition of limits. ■

3. Use the practical rules for computing limits to compute $\lim_{x \to -1} \frac{x^2-4x-5}{x^2-7x-8}$. *[5]*

SOLUTION. Observe that both the numerator and the denominator of the given rational function go to 0 as x goes to -1, since $(-1)^2 - 4 \cdot (-1) - 5 = 1 + 4 - 5 = 0$ and $(-1)^2 - 7 \cdot (-1) - 8 = 1 + 7 - 8 = 0$. One consequence is that $x - (-1) = x + 1$ is a factor of both the numerator and denominator; it shouldn't be hard to figure out that the other factors are $x - 5$ and $x - 8$, respectively. (Using the quadratic formula, if necessary.) It follows that

$$\lim_{x \to -1} \frac{x^2-4x-5}{x^2-7x-8} = \lim_{x \to -1} \frac{(x-5)(x+1)}{(x-8)(x+1)} = \lim_{x \to -1} \frac{x-5}{x-8} = \frac{-1-5}{-1-8} = \frac{-6}{-9} = \frac{2}{3}. \quad ■$$