Name: Solutions **Student Number:** ________________

Mathematics 1110H, TRENT UNIVERSITY, Fall 2025

Test #1 for Sections F02 and F04. *Friday, 26 September.* Time: 20 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do any two (2) of questions 1–3.** *If you do all three and don't cross one out, only the first two encountered by the graders will be marked.*

1. Find the x-intercepts and the location of the tip of the parabola $y = x^2 + 2x - 8$. *[5]*

SOLUTION I. *Intercepts first.* If you spot that $y = x^2 + 2x - 8 = (x-2)(x+4)$, it is easy to see that the x-intercepts of the parabola, the x values that make $y = 0$, are $x = 2$ and $x = -4$. If you don't spot the factorization, you can use the quadratic formula to find the x-intercepts:

$$
\begin{aligned}
y = x^2 + 2x - 8 = 0 \implies x &= \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-8)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4 - (-32)}}{2} \\
&= \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2} = \frac{4}{2} = 2 \text{ or } = \frac{-8}{2} = -4
\end{aligned}
$$

Either way, the x-coordinate of the tip of the parabola is halfway between the intercepts, *i.e.* $x = \dfrac{-4+2}{2} = \dfrac{-2}{2} = -1$. We plug this into the parabola's equation to get the y-coordinate of the tip: $y = (-1)^2 + 2(-1) - 8 = 1 - 2 - 8 = -9$ Thus the tip of the parabola is at the point $(-1, -9)$. □

SOLUTION II. *Tip first.* We shall find the coordinates of the tip of the parabola by completing the square in the equation defining the parabola:

$$
\begin{aligned}
y = x^2 + 2x - 8 &= \left(x + \frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 8 = (x+1)^2 - 1^2 - 8 \\
&= (x - (-1))^2 - 1 - 8 = (x - (-1))^2 - 9
\end{aligned}
$$

Thus the tip of the parabola is at the point $(-1, -9)$: the x-coordinate -1 makes the squared part 0, leaving -9 as the y-coordinate.

For the x-intercepts, we could factor the quadratic or apply the quadratic formula as in the solution above. Alternatively, we could observe that in the completed square form, $y = 0$ exactly when $(x - (-1))^2 - 9 = 0$, *i.e.* exactly when $(x+1)^2 = 9$. This means that $x + 1 = \pm\sqrt{9} = \pm 3$, so $x = -1 - 3 = -4$ or $x = -1 + 3 = 2$. Thus the x-intercepts of the parabola are $x = -4$ and $x = 2$. ■

2. Use the ε–δ definition of limits to verify that $\lim_{x \to -1} (2x+4) = 2$. *[5]*

SOLUTION. We need to check that for any given $\varepsilon > 0$, there is a $\delta > 0$, such that if $|x - (-1)| < \delta$, then $|(2x+4) - 2| < \varepsilon$. As usual we try to reverse-engineer the necessary δ from the desired conclusion:

$$|(2x+4) - 2| < \varepsilon \iff |2x+2| < \varepsilon \iff |2(x+1)| < \varepsilon \iff 2|x+1| < \varepsilon$$
$$\iff |x+1| < \frac{\varepsilon}{2} \iff |x - (-1)| < \frac{\varepsilon}{2}$$

If we set $\delta = \frac{\varepsilon}{2}$, then if $|x - (-1)| < \delta$, we are guaranteed to have $|(2x+4) - 2| < \varepsilon$, because every step in the reverse-engineering process above is fully reversible.

Since we can find a $\delta > 0$ for any $\varepsilon > 0$, $\lim_{x \to -1} (2x+4) = 2$ satisfies the ε–δ definition of limits. ■

3. Use the practical rules for computing limits to compute $\lim_{x \to 1} \frac{x^2 - 7x + 6}{x^2 + 3x - 4}$. *[5]*

SOLUTION. Observe that both the numerator and the denominator of the given rational function go to 0 as x goes to 1, since $1^2 - 7 \cdot 1 + 6 = 1 - 7 + 6 = 0$ and $1^2 + 3 \cdot 1 - 4 = 1 + 3 - 4 = 0$. One consequence is that $x - 1$ is a factor of both the numerator and denominator; it shouldn't be hard to figure out that the other factors are $x - 6$ and $x + 4$, respectively. (Using the quadratic formula, if necessary.) It follows that

$$\lim_{x \to 1} \frac{x^2 - 7x + 6}{x^2 + 3x - 4} = \lim_{x \to 1} \frac{(x-6)(x-1)}{(x+4)(x-1)} = \lim_{x \to 1} \frac{x-6}{x+4} = \frac{1-6}{1+4} = \frac{-5}{5} = -1. \quad ■$$