Name: Solutions **Student Number:** ____________

Mathematics 1110H, TRENT UNIVERSITY, Fall 2025

Test #1 for Section F01 and CAT. *Friday, 26 September.* Time: 20 minutes.

Instructions

- Write your name and student number at the top.
- Use only this sheet of paper, including the back side. If you need more paper, ask for it.
- You may use an aid sheet, A4- or letter-size with whatever you want written on all sides, and a calculator, with no restrictions beyond not being able to communicate with other devices.
- **Do any two (2) of questions 1–3.** *If you do all three and don't cross one out, only the first two encountered by the graders will be marked.*

1. Find the x-intercepts and the location of the tip of the parabola $y = x^2 + 6x + 5$. *[5]*

SOLUTION I. *Intercepts first.* If you spot that $y = x^2 + 6x + 5 = (x+1)(x+5)$, it is easy to see that the x-intercepts of the parabola, the x values that make $y = 0$, are $x = -1$ and $x = -5$. If you don't spot the factorization, you can use the quadratic formula to find the x-intercepts:

$$y = x^2 + 6x + 5 = 0 \implies x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{-6 \pm \sqrt{36 - 20}}{2}$$
$$= \frac{-6 \pm \sqrt{16}}{2} = \frac{-6 \pm 4}{2} = \frac{-2}{2} = -1 \text{ or } = \frac{-10}{2} = -5$$

Either way, the x-coordinate of the tip of the parabola is halfway between the intercepts, *i.e.* $x = \frac{-1-5}{2} = \frac{-6}{2} = -3$. We plug this into the parabola's equation to get the y-coordinate of the tip: $y = (-3)^2 + 6(-3) + 5 = 9 - 18 + 5 = -4$ Thus the tip of the parabola is at the point $(-3, -4)$. □

SOLUTION II. *Tip first.* We shall find the coordinates of the tip of the parabola by completing the square in the equation defining the parabola:

$$y = x^2 + 6x + 5 = \left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 5 = (x+3)^2 - 3^2 + 5$$
$$= (x - (-3))^2 - 9 + 5 = (x - (-3))^2 - 4$$

Thus the tip of the parabola is at the point $(-3, -4)$: the x-coordinate -3 makes the squared part 0, leaving -4 as the y-coordinate.

For the x-intercepts, we could factor the quadratic or apply the quadratic formula as in the solution above. Alternatively, we could observe that in the completed square form, $y = 0$ exactly when $(x - (-3))^2 - 4 = 0$, *i.e.* exactly when $(x+3)^2 = 4$. This means that $x + 3 = \pm\sqrt{4} = \pm 2$, so $x = 2 - 3 = -1$ or $x = -2 - 3 = -5$. Thus the x-intercepts of the parabola are $x = -1$ and $x = -5$. ■

2. Use the ε–δ definition of limits to verify that $\lim_{x \to 2}(3x+4) = 10$. *[5]*

SOLUTION. We need to check that for any given $\varepsilon > 0$, there is a $\delta > 0$, such that if $|x-2| < \delta$, then $|(3x+4)-10| < \varepsilon$. As usual we try to reverse-engineer the necessary δ from the desired conclusion:

$$\begin{aligned} |(3x+4)-10| < \varepsilon &\iff |3x-6| < \varepsilon \iff |3(x-2)| < \varepsilon \\ &\iff 3|x-2| < \varepsilon \iff |x-2| < \frac{\varepsilon}{3} \end{aligned}$$

If we set $\delta = \frac{\varepsilon}{3}$, then if $|x-2| < \delta$, we are guaranteed to have $|(3x+4)-10| < \varepsilon$, because every step in the reverse-engineering process above is fully reversible.

Since we can find a $\delta > 0$ for any $\varepsilon > 0$, $\lim_{x \to 2}(3x+4) = 10$ satisfies the ε–δ definition of limits. ■

3. Use the practical rules for computing limits to compute $\lim_{x \to 5} \frac{x^2-6x+5}{x^2-3x-10}$. *[5]*

SOLUTION. Observe that both the numerator and the denominator of the given rational function go to 0 as x goes to 5, since $5^2 - 6 \cdot 5 + 5 = 25 - 30 + 5 = 0$ and $5^2 - 3 \cdot 5 - 10 = 25 - 15 - 10 = 0$. One consequence is that $x-5$ is a factor of both the numerator and denominator; it shouldn't be hard to figure out that the other factors are $x-1$ and $x+2$, respectively. Using the quadratic formula, if necessary.) It follows that

$$\lim_{x \to 5} \frac{x^2-6x+5}{x^2-3x-10} = \lim_{x \to 5} \frac{(x-5)(x-1)}{(x-5)(x+2)} = \lim_{x \to 5} \frac{x-1}{x+2} = \frac{5-1}{5+2} = \frac{4}{7}. \quad ■$$