Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
Section A, Trent University, Fall 2025

Final Examination
11:00-14:00 on Monday, 8 December, in the Gym.

Instructions: Do both of parts **X** and **Y**, and, if you wish, part **Z**. Please show all your work, justify all your answers, and simplify these where you reasonably can. When you are asked to do k of n questions, only the first k that are not crossed out will be marked. *If you have a question, or are in doubt about something,* **ask!**

Aids: Any calculator, as long as it can't communicate with other devices; all sides of one letter- or A4-size sheet, with whatever you want written on it; your own brain.

Part X. Do all four (4) of **1**–**4**.

1. Compute $\dfrac{dy}{dx}$ as best you can in any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $y = \sqrt{1+x^4}$ **b.** $y = \dfrac{x+1}{x-1}$ **c.** $y = \left(e^x - e^{-x}\right)^2$

d. $y^2 - x^2 = 1$ **e.** $y = \ln\left(x^{41}\right)$ **f.** $y = \sec(x)\tan(x)$

2. Evaluate any four (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int_0^2 (x-2)^2\,dx$ **b.** $\displaystyle\int (x\ln(x))^2\,dx$ **c.** $\displaystyle\int_0^{\pi/2} x\cos(x)\,dx$

d. $\displaystyle\int 2xe^{x^2}\,dx$ **e.** $\displaystyle\int_0^{\pi} 2\sin(x)\cos(x)\,dx$ **f.** $\displaystyle\int x\sqrt{x^2+4}\,dx$

3. Do any four (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Find the area between $y = \sqrt{x}$ and $y = \frac{x}{2}$, where $0 \le x \le 4$.

b. Use the ε–δ definition of limits to verify that $\displaystyle\lim_{x\to 4}(3x-11) = 1$.

c. Compute $\displaystyle\lim_{x\to\infty}\frac{x^2}{2+3x^2}$.

d. Find the volume of the solid obtained by revolving the region between the line $x = 1$ and the line $y = x$, for $0 \le y \le 1$, about the x-axis.

e. Use the limit definition of the derivative to compute $\dfrac{d}{dx}(2x+3)$.

f. Determine whether $f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous at $x = 0$ or not.

4. Find the domain, intercepts, vertical and horizontal asymptotes, intervals of increase and decrease, maximum and minimum points, intervals of concavity, and inflection points of $f(x) = \dfrac{x^2}{1+x^2}$, and sketch its graph based on this information. *[14]*

There is more on page 2!

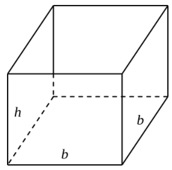
Part Y. Do any two (2) of **5**–**7**. *[26 = 2 × 13 each]* *Here is the "more"!*

5. The region below $y = \sqrt{x-1}$ and above $y = 0$, where $1 \le x \le 5$, is revolved about the y-axis, making a solid of revolution.

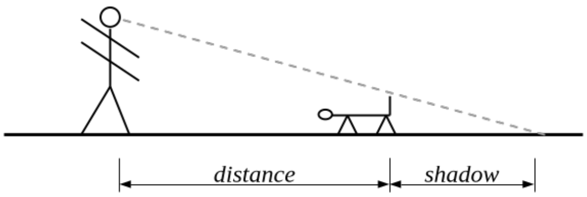
a. Sketch the region. *[1]* **b.** Find the area of the region. *[3]*

c. Sketch the solid. *[1]* **d.** Find the volume of the solid. *[8]*

6. A small cardboard box has a square bottom and no top. If 48 cm^2 of cardboard are used to make the box, what is its maximum possible volume? What are the dimensions of such a box of maximum volume? *[13]*



7. It is night in a dark and narrow alley. A four-armed robot, bearing a headlight 1.2 m above the pavement, moves along the alley at 1 m/s from one end, and a kitten, holding the tip of its straight-up tail 0.4 m above the pavement, moves along the alley at 1 m/s from its other end. How is the length of the shadow cast by the kitten's rear and tail changing at the instant that the robot and the kitten are 4 m apart? *[13]*



[Total = 100]

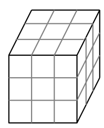
Part Z. *Bonus points!* Do one or both of **8** and **9**.

8. Write an original haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

9. A dangerously sharp tool is used to cut a cube with a side length of 3 cm into 27 smaller cubes with a side length of 1 cm. This can be done easily with six cuts. Can it be done with fewer? (Rearranging the pieces between cuts is allowed.) If so, explain how; if not, explain why not. *[1]*



APOLOGIES FOR ALL THE GLITCHES.
HAVE A GOOD BREAK!