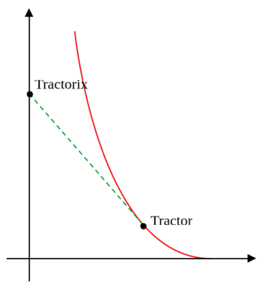
Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals
TRENT UNIVERSITY, Fall 2025

Solutions to Assignment #4
Tractor Pull Curve?
Due on Friday, 7 November.

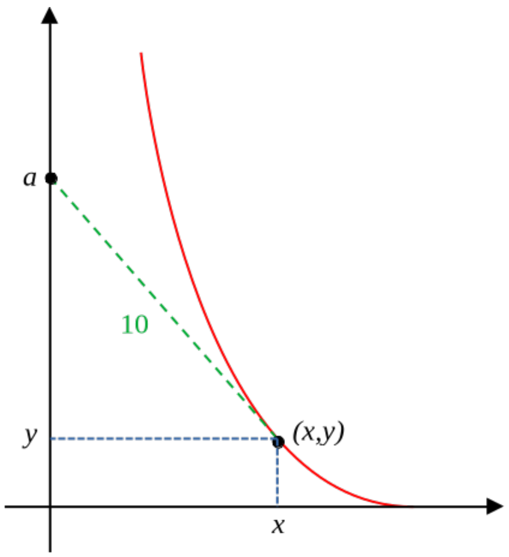
A modern relative of Obelix†, Tractorix, is pulling a tractor in the Cartesian plane using a 10 m cable. At the beginning, Tractorix is at the origin and the tractor is 10 m away on the positive x-axis. Tractorix then begins to walk up the positive y-axis, pulling the tractor along. At each instant, the cable is straight and taut, without stretching, and is tangent to the curve the tractor is dragged along.



1. If (x, y) is a point on the curve the tractor is being dragged along, find $\frac{dy}{dx}$ as a function of x. *[5]*

Hint: If Tractorix is at $(0, a)$ on the y-axis at the instant that the tractor is at (x, y), then the distance between them is 10, but can also be computed another way. Similarly, the slope at that instant of the curve the tractor is being dragged along can be computed in two different ways.

SOLUTION. Consider the following annotated version of the diagram for this setup.



† With apologies to René Goscinny (1926-1977) and Albert Uderzo (1927-2020), the creators of the comic *Asterix*, in which Obelix is a character.

The tractor is at (x, y) and Tractorix is at $(0, a)$ on the y-axis at the given instant. Observe that these two points, along with the point $(0, y)$ on the y-axis, are the vertices of a right triangle with short sides of length x and $a - y$ and a hypotenuse of length 10.

The slope of the curve at (x, y) is given by $\frac{dy}{dx}$ on the one hand. On the other hand, since the leash – *i.e.* the hypotenuse of the triangle – is tangent to the curve, it is also given by $\frac{\Delta y}{\Delta x} = \frac{y-a}{x-0} = \frac{y-a}{x}$. By the Pythagorean Theorem, we have $x^2 + (a-y)^2 = 10^2 = 100$, from which it follows that $y - a = -\sqrt{100 - x^2}$. (We take the negative root because $y < a$ in the setup.) Putting all this together, we get that $\frac{dy}{dx} = \frac{y-a}{x} = -\frac{\sqrt{100-x^2}}{x}$. *Per* original setup, we also have the initial condition that $y = 0$ when $x = 10$. ■

2. Use SageMath to solve the differential equation you obtained in solving **1**. (Initial conditions included!) *[3]*

Solution. Here we go:

```
[1]: y = function('y')(x)
     desolve( diff(y,x) == -sqrt(100 - x^2)/x, y, ics=[10,0] )
```

```
[1]: -sqrt(-x^2 + 100) - 10*log(20) + 10*log(20*(sqrt(-x^2 + 100) + 10)/abs(x))
```

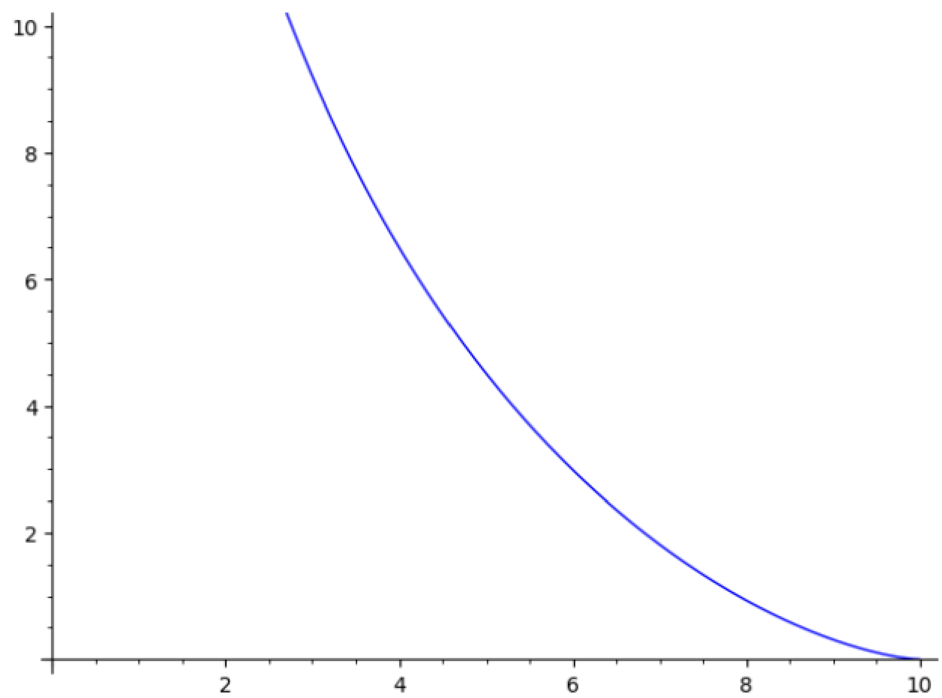
■

3. Use SageMath to plot the curve the tractor is dragged along for $0 \leq y \leq 10$. *[1]*

Solution. Here we go:

```
[2]: plot( -sqrt(-x^2 + 100) - 10*log(20) + 10*log(20*(sqrt(-x^2 + 100) + 10)/
     ↪abs(x)), 0, 10, ymin=0, ymax=10 )
```

[2]:



■

4. Assuming Tractorix keeps walking up the y-axis and continues to drag the tractor, does the tractor ever reach the y-axis? Explain why or why not. *[1]*

SOLUTION. The tractor, or at least the moving point representing it in this problem, never reaches the y-axis, no matter how far along it Tractorix walks. The function we obtained in **2** involves a division by $|x|$, which would be a bit awkward on the y-axis, otherwise known as the line $x = 0$ . . . $\square$