## Mathematics 1110H - Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Fall 2025

## Assignment #2

## Finding Limits and Solving Equations With SageMath

Due on Friday, 3 October.\*

Please at least skim through Section 1.8, especially subsections 1.8.1 and 1.8.4, and Section 4.8 of Gregory Bard's  $Sage\ for\ Undergraduates$ , or go to the labs, before tackling this assignment.

1. Use SageMath's limit command to compute each of the following limits.

**a.** 
$$\lim_{x \to 0} \frac{e^{-1/x^2}}{x^2}$$
 [1]

**b.** 
$$\lim_{x\to\infty} (\pi - \arctan(x))$$
 [1]

c. 
$$\lim_{x \to 0} (\cos(x))^{1/x^2}$$
 [1]

2. Use SageMath's solve command in parts a-c to ...

**a.** solve for 
$$x$$
 in  $x^2 + 2x = 3$ .  $[0.5]$ 

**b.** solve for 
$$x$$
 in  $x^3 + 3x^2 - x = 3$ . [0.5]

- **c.** solve for x and y in the system of equations  $x^2 y^2 = 1$  and  $x^2 + y^2 = 4$ . [1]
- **d.** Plot the curves  $x^2 y^2 = 1$  and  $x^2 + y^2 = 4$  together in different colours for  $-3 \le x \le 3$  and  $-3 \le y \le 3$ . [0.5]

**3.** Consider the hyperbola  $y^2 - x^2 = 1$ . (It's not the same as the one in **2c** and **2d**.)

- **a.** Plot the hyperbola and the lines y = x/2, y = x, and y = 2x together in different colours for  $-5 \le x \le 5$ . [1]
- **b.** Use the solve command to find the point(s) of intersection, if any, of the hyperbola with each of the lines in part **a**.  $[3 \times 0.5 \ each = 1.5]$
- c. Use the limit command to compute each of the limits  $\lim_{x\to\infty} \left(x-\sqrt{1+x^2}\right)$  and  $\lim_{x\to-\infty} \left(x+\sqrt{1+x^2}\right)$ . [1]
- **d.** What do parts **a**–**c** tell you about the relationship between the given hyperbola and the line y = x? [1]

|Total = 10|

<sup>\*</sup> You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you write up your submission by yourself and give due credit to your collaborators and any sources and tools you actually used.