MATH1110H-A2-Solutions

September 16, 2025

[1]:
```
# MATH 1110H,  Fall 2025
# Solutions to Assignment #2
```

[2]:
```
# 1a
limit( e^(-1/x^2) / x^2, x=0 )
```

[2]: 0

[3]:
```
# 1b
limit( pi - arctan(x), x=oo )
```

[3]: 1/2*pi

[4]:
```
# 1c
limit((cos(x))^(1/x^2), x=0 )
```

[4]: e^(-1/2)

[5]:
```
# 2a
solve( x^2 + 2*x == 3, x )
```

[5]: [x == -3, x == 1]

[6]:
```
# 2b
solve( x^3 + 3*x^2 - x == 3, x )
```

[6]: [x == 1, x == -1, x == -3]

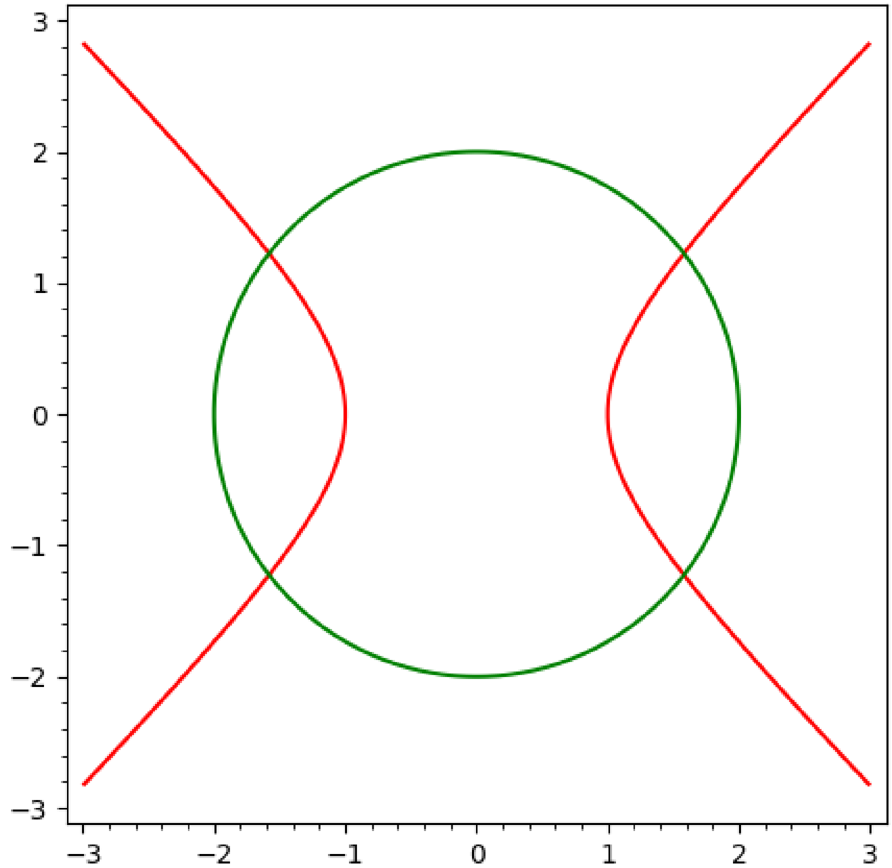
[7]:
```
# 2c
var('y')
solve( [x^2 - y^2 == 1, x^2 + y^2 == 4], x, y )
```

[7]:
```
[[x == -1/2*sqrt(5)*sqrt(2), y == -1/2*sqrt(3)*sqrt(2)], [x ==
-1/2*sqrt(5)*sqrt(2), y == 1/2*sqrt(3)*sqrt(2)], [x == 1/2*sqrt(5)*sqrt(2), y ==
-1/2*sqrt(3)*sqrt(2)], [x == 1/2*sqrt(5)*sqrt(2), y == 1/2*sqrt(3)*sqrt(2)]]
```

[8]:
```
# 2d
g1 = implicit_plot( x^2 - y^2 == 1, (x,-3,3), (y,-3,3), color='red' )
g2 = implicit_plot( x^2 + y^2 == 4, (x,-3,3), (y,-3,3), color='green' )
```

```
g1 + g2
```

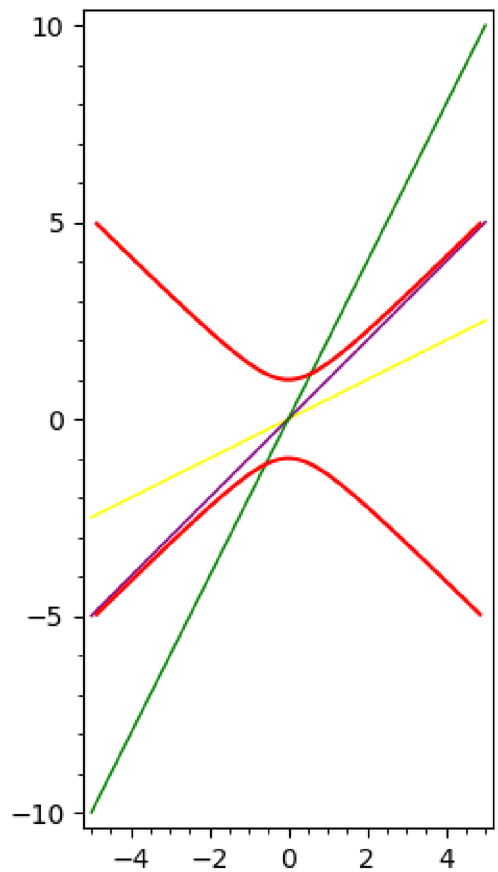
[8]:



[9]:

```
# 3a
p1 = plot( x/2, x, -5, 5, color='yellow' )
p2 = plot( x, x, -5, 5, color='purple' )
p3 = plot( 2*x, x, -5, 5, color='green' )
p4 = implicit_plot( y^2 - x^2 == 1, (x,-5,5), (y,-5,5), color='red')
p1 + p2 + p3 + p4
```

[9]:



[10]:
```
# 3b
show(solve( [y^2 - x^2 == 1, y == x/2], x, y ))
show(solve( [y^2 - x^2 == 1, y == x], x, y ))
show(solve( [y^2 - x^2 == 1, y == 2*x], x, y ))
```

$$\left[\left[x = -\frac{2}{3}i\,\sqrt{3}, y = -\frac{1}{3}i\,\sqrt{3}\right], \left[x = \frac{2}{3}i\,\sqrt{3}, y = \frac{1}{3}i\,\sqrt{3}\right]\right]$$

$$[]$$

$$\left[\left[x = -\frac{1}{3}\,\sqrt{3}, y = -\frac{2}{3}\,\sqrt{3}\right], \left[x = \frac{1}{3}\,\sqrt{3}, y = \frac{2}{3}\,\sqrt{3}\right]\right]$$

[11]:
```
# Note that y^2 - x^2 = 1 and y = x/2 do not intersect because the
# only common solutions of the two equations are complex numbers.
# y^2 - x^2 = 1 and y = x also don't intersect, as the two equations
# don't even have complex numbers as common solutions.  On the other
# hand,  y^2 - x^2 = 1 and y = 2x do intersect as the two equations
```

```
# common real solutions.
```

[12]:
```
# 3c
show(limit( x - sqrt(1+x^2), x=oo ))
show(limit( x - (-sqrt(1+x^2)), x=-oo ))
```

$$0$$

$$0$$

[13]:
```
# 3d
#
# Note that y = sqrt(1+x^2) is the upper part of the given hyperbola
# and y = - sqrt(1+x^2) is the lower part of the given hyperbola.
#
# Question 3b tells us that the given hyperbola and the line y = x
# never intersect, while question 3c tells us that the upper part
# of the hyperbola gets arbitrarily close to the line as x goes to
# infinity, and that the lower part of the hyperbola also gets
# arbitrarily close to the line, but as x goes to negative infinity.
# Note that this is consistent with the plot in question 3a.

# The line y = x is thus a "slant asymptote" of the hyperbola, as
# is, by the way, the line y = -x.
```