Calculus I : Sep 8

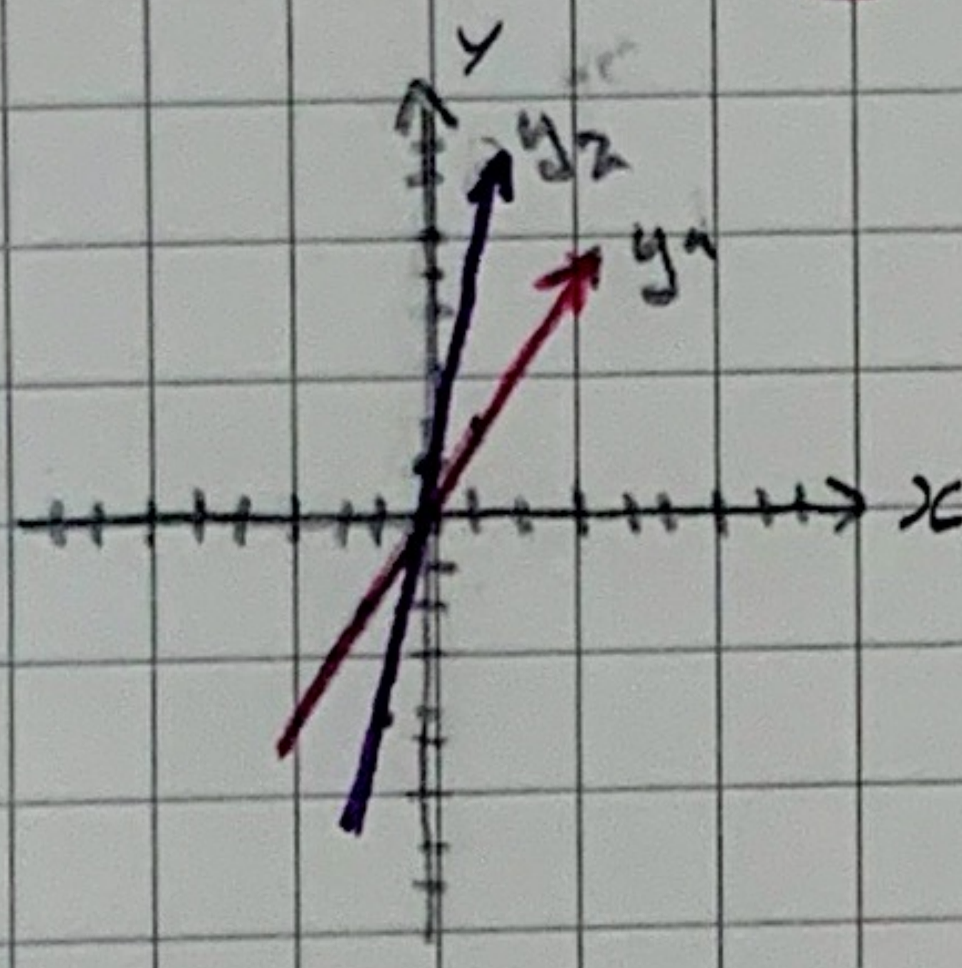
Primes (Real) variable

a b c d e f g h i j k l m n o p q r s t u v w x y z

constants | function | Integer variable | Rational variable | Various Greek letters

Connection between graphs & algebra
Cartesian coordinate system

Linear: $y_1 = 2x$ $\quad y_2 = 5x + 1$



How to find the point of intersection

$$\left.\begin{aligned}
&\Rightarrow 2x = y = 5x + 1 \\
&\Rightarrow y = 5x - 2x + 1 \\
&\Rightarrow y = 3x + 1 \\
&\Rightarrow 0 = 3x + 1 \\
&\Rightarrow -1 = 3x \\
&\Rightarrow -1/3 = x
\end{aligned}\right\} \text{finding } x$$

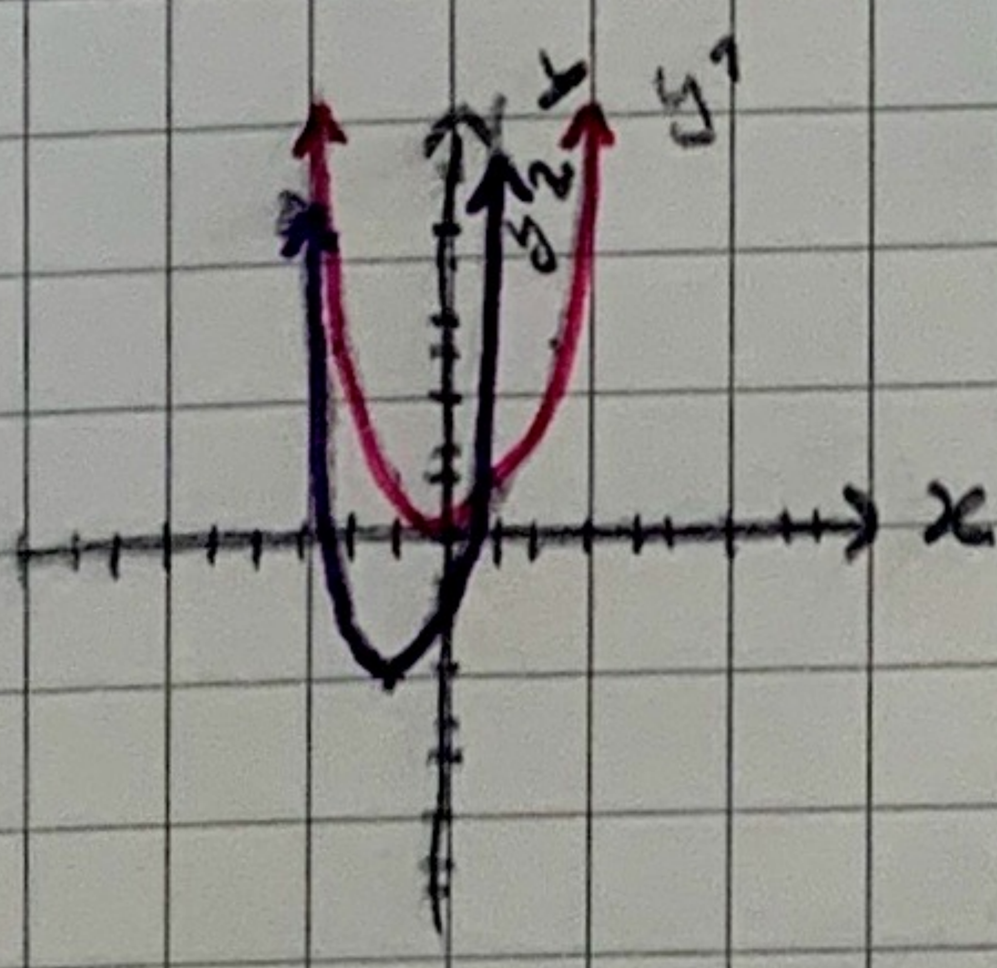
$\Rightarrow y = 2(-1/3)$

$\Rightarrow y = -2/3$

P.I. : $(-1/3, -2/3)$

Parabola: $y_1 = x^2$ $\quad y_2 = 2x^2 + 4x - 1$

↳ How to find the tip
$\Rightarrow$ Between the two x-ints



$\Rightarrow$ y-int : $(x = 0)$

$\Rightarrow y = 2(0)^2 + 4(0) - 1$

$\Rightarrow y = 0 + 0 - 1$

$\Rightarrow y = -1$

$\Rightarrow$ x-int : $(y = 0)$
↳ Factor
↳ Quadratic equation

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-1)}}{2(2)}$$

$$\Rightarrow \frac{-4 \pm \sqrt{24}}{4} \, ^*$$

$$\Rightarrow \frac{-4}{4} \pm \frac{2\sqrt{6}}{4}$$

$$\Rightarrow -1 \pm \frac{\sqrt{6}}{2} = x_1, x_2$$

$^* \sqrt{24} = \sqrt{8 \cdot 3}$

$= \sqrt{4 \cdot 2 \cdot 3} = 2\sqrt{6}$

Tip of y_2 : $(-1, -3)$ ☆

x: between $-1+\frac{\sqrt{6}}{2}$ & $-1-\frac{\sqrt{6}}{2}$

$x = -1$

$y = 2(-1)^2 + 4(-1) - 1$

$y = 2 - 4 - 1$

$y = -3$

What if you don't have intercepts?

Complete the square

Ex: $y = ax^2 + bx + c \quad (a \neq 0)$

$y = a\left(x^2 + \frac{bx}{a} + \frac{c}{a}\right)$ Factor out a, leave aside $\frac{c}{a}$

Taking $x^2 + \frac{b}{a}x$:

$(x^2 + k)^2 = x^2 + 2kx + k^2$

$2k = \frac{b}{a} \Rightarrow k = \frac{b}{2a}$

$$y = a\Big(\underbrace{\left[x + \frac{b}{2a}\right]^2}_{x^2 + \frac{bx}{a} + \frac{b^2}{4a}} - \frac{b^2}{4a} + \frac{c}{a}\Big)$$

to cancel out

$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

↳ If a is positive ∪

If a is negative ∩

The biggest or smallest value of y occurs when $\left(x + \frac{b}{2a}\right)^2 = 0$ so $x = \frac{-b}{2a}$

$y = a(0)^2 - \frac{b^2}{4a} + c \Rightarrow y = \frac{-b^2}{4a} + c$

Tip of $y = ax^2 + bx + c$:

$\left(\frac{-b}{2a}, \frac{-b^2}{4a} + c\right)$ ☆

Deriving the quadratic formula

$$a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c=0$$

$$a\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a}-c$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}$$

$$x=\frac{-b}{2a}\pm\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}$$

$$x=\frac{-b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$