Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals (Section C)
TRENT UNIVERSITY, Fall 2021

Solutions to Quiz #2
Wednesday, 29 September.

Do both of the following problems.

1. Use the practical rules for derivatives to compute the derivative of $f(x) = \dfrac{1}{\sqrt{x}}$. *[1]*

SOLUTION. We will use the Power Rule for derivatives as our principal tool:

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{d}{dx}x^{-1/2} = -\frac{1}{2} \cdot x^{-\frac{1}{2}-1} = -\frac{1}{2} \cdot x^{-3/2} = \frac{-1}{2x^3/2} = \frac{-1}{2x\sqrt{x}} \quad \blacksquare$$

2. Use the limit definition of the derivative to find the derivative of $f(x) = \dfrac{1}{\sqrt{x}}$. *[4]*

SOLUTION. We will plug the given function into the limit definition of the derivative and hope for the best:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h \to 0} \frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x+h}\sqrt{x}}}{h} = \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} = \lim_{h \to 0} \frac{\left(\sqrt{(x)}\right)^2 - \left(\sqrt{x+h}\right)^2}{h\sqrt{x+h}\sqrt{x}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{h\sqrt{x+h}\sqrt{x}\left(\sqrt{x} + \sqrt{x+h}\right)} = \lim_{h \to 0} \frac{-h}{h\sqrt{x+h}\sqrt{x}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$\lim_{h \to 0} \frac{-1}{\sqrt{x+h}\sqrt{x}\left(\sqrt{x} + \sqrt{x+h}\right)} = \frac{-1}{\sqrt{x+0}\sqrt{x}\left(\sqrt{x} + \sqrt{x+0}\right)}$$

$$= \frac{-1}{\sqrt{x}\sqrt{x}\left(\sqrt{x} + \sqrt{x}\right)} = \frac{-1}{x \cdot 2\sqrt{x}} = \frac{-1}{2x\sqrt{x}}$$

Whew! Fortunately, this is the same answer we obtained in **1** above ... :-) $\blacksquare$

[Total = 5]