# Integration by Parts II - Dividing up the integrand and other tricks

Recap: $\int u \cdot v' dx = uv - \int u' \cdot v \, dx$ $\quad$ [Converse to $(uv)' = u' \cdot v + u \cdot v'$]

Integration by Parts

Q.: How do divide up a given integrand into $u$ & $v'$?

2 Rules of Thumb [neither works all the time, either one could give integrals that require additional tricks]

1° Choose $u$ and $v'$ so that $\int u' \cdot v \, dx$ is simpler.

eg $\int x e^x dx$ $\qquad$ $u = x \quad v' = e^x$

$u' = 1 \quad v = e^x$

$= xe^x - \int e^x dx$ ... which is simpler.

2° If you have an integrand that is the product of two dissimilar functions, put whicher appears first the list below into $u$ and the rest into $v'$.

List: logarithmic (and inverse hyperbolic)

inverse trigonometric

polynomials (and others not elsewhere on this list)

trigonometric

exponential (and hyperbolic)

Ex $\int x e^x \, dx$ $\qquad$ $x$ is a polynomial & $e^x$ is exponential

so put $x$ into $u$ & $e^x$ into $v'$

ie $\quad u = x \qquad v' = e^x$

$\quad u' = 1 \qquad v = e^x$

$$= x e^x - \int 1 e^x \, dx \quad \text{etc}$$

eg $\int x \ln(1+x^2)\,dx$

$x$ is a polynomial
& $\ln(1+x^2)$ is a logarithm

$= uv - \int u'v\,dx$

so try $u = \ln(1+x^2)$ & $v' = x$

so $u' = \frac{1}{1+x^2} \cdot 2x$ & $v = \frac{x^2}{2}$

$$= \frac{x^2}{2}\ln(1+x^2) - \int \frac{\cancel{2}x}{1+x^2} \cdot \frac{x^2}{\cancel{2}}\,dx = \frac{x^2}{2}\ln(x^2+1) - \int \frac{x^3}{x^2+1}\,dx$$

Use $x^3 = x(x^2+1) - x$

$$= \frac{x^2}{2} \cdot \ln(1+x^2) - \int \frac{x(1+x^2) - x}{1+x^2}\,dx = \frac{x^2}{2}\ln(1+x^2) - \int \frac{x\cancel{(1+x^2)}}{\cancel{1+x^2}}\,dx + \int \frac{x}{1+x^2}\,dx$$

$w = 1+x^2$
$dw = 2x\,dx$
$x\,dx = \frac{1}{2}dw$

$$= \frac{x^2}{2}\ln(1+x^2) - \int x\,dx + \int \frac{1}{w} \cdot \frac{1}{2}\,dw$$

$$= \frac{x^2}{2}\ln(1+x^2) - \frac{x^2}{2} + \frac{1}{2}\ln(w) + C$$

$$= \frac{x^2}{2}\ln(1+x^2) - \frac{x^2}{2} + \frac{1}{2}\ln(1+x^2) + C$$

eg $\boxed{\int e^x \cos(x)\,dx}$

Since trig functions occur before exponentials on the list, we should try

$u = \cos(x)$ & $v' = e^x$, first.

[So $u' = -\sin(x)$ & $v = e^x$ ]

$= uv - \int u' \cdot v\,dx$

$= e^x\cos(x) - \int(-\sin(x))e^x\,dx = e^x\cos(x) + \int e^x \sin(x)\,dx$

Since trig comes before exponential on the list try

$s = \sin(x)$ & $t' = e^x$,

so $s' = +\cos(x)$ & $t = e^x$.

$= e^x\cos(x) + e^x\sin(x) - \int(+\cos(x))e^x\,dx$

$= e^x[\cos(x) + \sin(x)] - \boxed{\int e^x \cos(x)\,dx}$

& Solve for the boxed integral...

$\Rightarrow 2\boxed{\int e^x\cos(x)\,dx} = e^x(\cos(x) + \sin(x))$

$\Rightarrow \int e^x \cos(x)\,dx = \frac{1}{2}e^x(\cos(x) + \sin(x)) + C$

More examples & some applications next time.