**Mathematics 1110H – Calculus I: Limits, Derivatives, and Integrals**
TRENT UNIVERSITY, Fall 2018

**Solutions to Assignment #5**
**`Maple` Differentiates**

The focus of this assignment is to play a little with what *Maple* can do with taking and manipulating derivatives.

**1.** Use *Maple* to find all the points where the graph of $p(x) = 5x^5 + 4x^4 + 3x^3 + 2x^2 + x$ has slope 0, without taking the derivative of $p(x)$ by hand. *[2]*

SOLUTION. First, we have `Maple` compute the derivative of $p(x)$:

```
> f := diff( 5*x^5 + 4*x^4 + 3*x^3 + 2*x^2 + x, x)
```

$$f := 25x^4 + 16x^3 + 9x^2 + 4x + 1$$

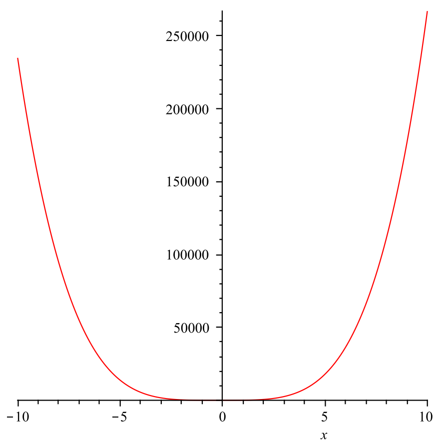
Then we have `Maple` determine where the derivative is 0:

```
> solve(f=0)
```

$$RootOf\left(25\_Z^4 + 16\_Z^3 + 9\_Z^2 + 4\_Z + 1, index = 1\right), RootOf\left(25\_Z^4 + 16\_Z^3 + 9\_Z^2 + 4\_Z + 1, index = 2\right), RootOf\left(25\_Z^4 + 16\_Z^3 + 9\_Z^2 + 4\_Z + 1, index = 3\right), RootOf\left(25\_Z^4 + 16\_Z^3 + 9\_Z^2 + 4\_Z + 1, index = 4\right)$$

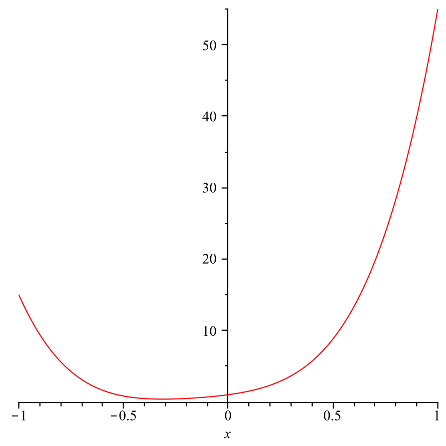
This gibberish is a clue that there are no real values of $x$ for which the derivative is 0. We can't be completely sure, though, since `Maple` will sometimes toss out stuff like this when there are real solutions, so we plot the derivative to see if ever has $y = 0$, *i.e.* whether it touches the $x$-axis.

```
> plot(f)
```




Hmm – it seems to get really close to the $x$-axis near 0, so we'll rescale the graph to get a better idea of what happens there:

> `plot(f,x=-1..1)`




It seems that it doesn't touch the $x$-axis after all, though it does get very close to it near $x = -0.3$ or so. Thus $p(x) = 5x^5 + 4x^4 + 3x^3 + 2x^2 + x$ never has slope 0. ■

**2.** Use *Maple* to help determine which of the points from **1** are maxima (peaks), minima (valleys), or neither of the graph of $p(x)$. *[2]*

Solution. As no points where the slope is 0 were found in question **1**, there is nothing to do here. :-) In particular, note that because $p(x)$, like every polynomial, is defined, continuous, and differentiable for all $x$, the fact that it never has slope 0 means that it has no maxima or minima. ■

A differential equation is an equation in which the derivative(s) of some unknown function(s) appear. The usual task is to find the unknown functions that satisfy the equation; this normally requires some additional information about specific values of the function(s) and/or the derivative(s) at specific points in order to fully pin down the unknowns.

Consider the differential equation $\frac{dy}{dx} = e^{x+y}$, with initial condition $y(0) = 0$ (*i.e.* with $y = 0$ when $x = 0$). A solution to this differential equation with the given initial condition would be a function $y = f(x)$ that satisfies both the equation, *i.e.* such that $f'(x) = e^{x+f(x)}$, and the given initial condition, *i.e.* such that $f(0) = 0$. `Maple` has ways of finding such solutions ...

**3.** Use `Maple` to find (all the) solution(s) of this differential equation with the given initial condition. Plot your solution(s) and figure out the(ir) domain and range. *[3]*

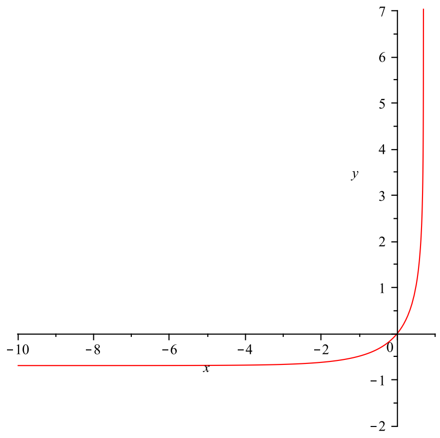
Solution. We will use the `dsolve` command to find the solution to the given differential equation and initial condition:

> `dsolve( {diff(y(x),x) = exp(x+y(x)), y(0) = 0}, y(x))`

$$y(x) = -\ln\left(-e^x + 2\right)$$

Now for the plot:

```
> plot( -ln(-exp(x)+2), x=-10..1, y=-2..7 )
```




It remains to find the domain and range of this solution, which we will do partly by hand and partly by using `Maple`. For the domain, note that $y(x) = -\ln(-e^x + 2)$ is defined whenever the input of the natural logarithm, $-e^x + 2$, is positive. Since $-e^x + 2 > 0$ exactly when $e^x < 2$, which happens when $x < \ln(2)$, it follows that the domain of the solution $y(x) = -\ln(-e^x + 2)$ is $(-\infty, \ln(2))$.

The plot suggests that the function has a horizontal asymptote as $x \to -\infty$ and a vertical asymptote as $x \to \ln(2)^-$. We use `Maple`'s `limit` operator to check this:

```
> limit( -ln(-exp(x)+2), x=-infinity )
```

$$-\ln(2)$$

```
> limit( -ln(-exp(x)+2), x=ln(2), left )
```

$$\infty$$

Since it is clear from the plot that the range of $y(x)$ falls between the values given by the horizontal and vertical asymptotes, this range must be $(-\ln(2), \infty)$. ■

**4.** Use `Maple` to find (all the) solution(s) of this differential equation with the initial condition $y(0) = 1$ instead. Plot your solution(s) and figure out the(ir) domain and range. *[3]*

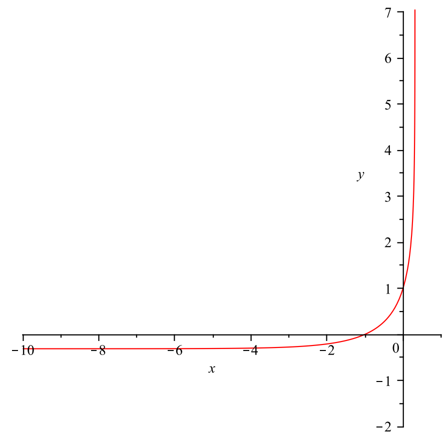
Solution. Just as in the solution to **3**, we will use the `dsolve` command to find the solution to the given differential equation and initial condition:

```
> dsolve( {diff(y(x),x) = exp(x+y(x)), y(0) = 1}, y(x))
```

$$y(x) = -\ln\left(-\frac{e^x e - e - 1}{e}\right)$$

Now for the plot:

$>$ `plot( -ln( -(exp(x)*exp(1) - exp(1) - 1)/exp(1) ), x=-10..1, y=-2..7 )`




It looks like the curve as in **3**, just shifted a bit to the left and up, which means that it should have a horizontal and vertical asymptote, just shifted a bit. Again, it remains to find the domain and range of this solution, which we will again do partly by hand and partly by using `Maple`.

For the domain, note that $y(x) = -\ln\left(-\dfrac{e^x e - e - 1}{e}\right)$ is defined whenever the input of the natural logarithm, $-\frac{e^x e - e - 1}{e}$, is positive. Since $-\frac{e^x e - e - 1}{e} > 0$ exactly when $e^x e = e^{x+1} < e + 1$, which happens when $x < \ln(e+1) - 1$, it follows that the domain of the solution $y(x)$ is $(-\infty, \ln(e+1) - 1)$.

As we did in the solution to **3**, we use `Maple`'s `limit` operator to check for the horizontal and vertical asymptotes suggested by the plot:

$>$ `limit( -ln( -(exp(x)*exp(1) - exp(1) - 1)/exp(1), x=-infinity )`

$$-\ln(e+1) + 1$$

$>$ `limit( -ln( -(exp(x)*exp(1) - exp(1) - 1)/exp(1), x=ln(exp(1)+1)-1, left )`

$$\infty$$

It follows that the range of $y(x)$ is $(-\ln(e+1) + 1, \infty)$. ■