Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
TRENT UNIVERSITY, Fall 2018
The Final ~~Countdown~~ Examination

Space-time: *Gym – 11:00-14:00.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts □ and △, and, if you wish, part ○. Please show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Any calculator; (all sides of) one aid sheet; one (1) brain (no neuron limit).

Part □. Do all four (4) of **7**–**4**. *[Subtotal = 74]*

7. Compute $\frac{dy}{dx}$ as best you can in any *four* (4) of **f**–**a**. *[20 = 4 × 5 each]*

f. $y = (x-1)e^x$ **e.** $\ln(y-x) = 0$ **d.** $y = \int_0^{\sqrt{x}} \cos(t)\,dt$

c. $y = \frac{x-1}{x+1}$ **b.** $y = \tan\left(e^{2x}\right)$ **a.** $y = x^\pi + x^e + 2018$

6. Evaluate any *four* (4) of the integrals **f**–**a**. *[20 = 4 × 5 each]*

f. $\int e^x \cos(x)\,dx$ **e.** $\int_1^2 \frac{w^2-w-2}{w+1}\,dw$ **d.** $\int_0^{\pi/4} 2\tan(z)\sec^2(z)\,dz$

c. $\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy$ **b.** $\int t\cosh(t)\,dt$ **a.** $\int_0^{\pi/2} \frac{\cos(u)}{1+\sin^2(u)}\,du$

5. Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

f. Find the equation of the tangent line to $y = \tan(x)$ at $x = 0$.

e. Use the ε–δ definition of limits to verify that $\lim_{x \to -1}(2x+3) = 1$.

d. Use the limit definition of the derivative to verify that $\frac{d}{dx}x^3 = 3x^2$ for all x.

c. Find the minimum value of $f(x) = x^3 - x^2 + x$ on the interval $[0,2]$.

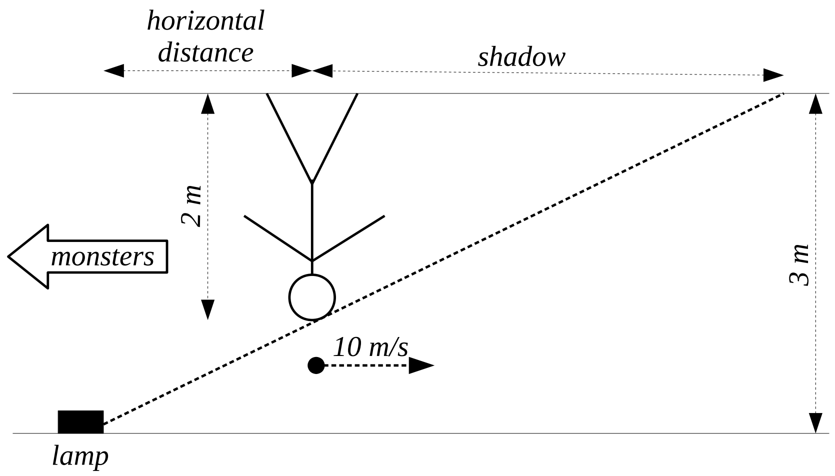
b. Compute $\lim_{x \to \infty} \frac{e^x}{e^{2x}+1}$.

a. Find the area of the the region between $y = \cos(x)$ and $y = \sin(x)$ for $0 \le x \le \frac{\pi}{2}$.

4. Find the domain and any and all intercepts, vertical and horizontal asymptotes, intervals of increase, decrease and concavity, and maximum, minimum, and inflection points of $f(x) = \frac{x^2}{x^2+1}$, and sketch its graph. *[14]*

More on the next page!

Part △. Do any *two* (2) of **3**–**1**. *[Subtotal = 26]*



3. Jed Aye, who is 2 m tall, explores a 3 m tall tunnel that runs horizontally, walking on the ceiling to avoid traps and carrying a lamp. On spotting monsters farther down the tunnel, Jed drops the lamp in shock and awe and begins running away from the monsters, still on the ceiling, at 10 m/s. At the instant that Jed is a horizontal distance of 10 m away from where the still-functioning lamp landed on the floor, how is the length of Jed's shadow on the ceiling changing with time? *[13]*

2. What is the maximum area of a rectangle which has each side parallel to one of the axes and all of its corners on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$? *[13]*

1. Sketch the finite region between $y = f(x)$ and $y = g(x)$ and find its area for:

b. $f(x) = \sin(x)$ and $g(x) = \frac{2x}{\pi}$. *[6]* **a.** $f(x) = \sin^2(x)$ and $g(x) = \frac{4x^2}{\pi^2}$. *[7]*

[Total = 100]

Part ○. Bonus problems! If you feel like it and have the time, do one or both of these.

⊚. The longest straight line that fits inside a perfectly circular road of constant width is 100 m long. What is the area covered by the road? *[1]*

⊙. Write a haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

Enjoy the break!