

Mathematics 1101Y – Calculus I: Functions and calculus of one variable

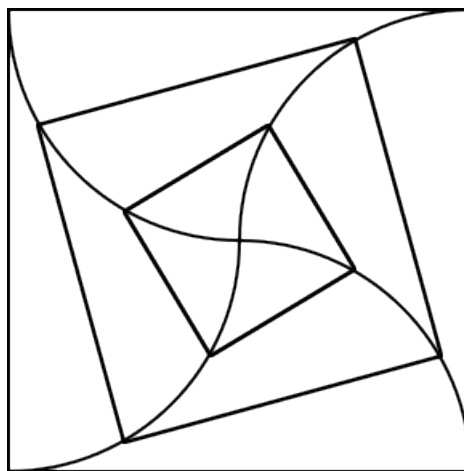
TRENT UNIVERSITY, 2012–2013

Assignment #3

The Beetles

Due on Friday, 11 January, 2013.

Four beetles are placed on the corners of a square with sides of length 10 *cm*. Simultaneously, each beetle begins to crawl straight towards the next beetle in the next (considered counterclockwise) corner. Each beetle crawls at the same constant rate as every other beetle and at every instant it crawls directly towards its target beetle, so the locations of the beetles form a square at every instant. Eventually, of course, they all meet at the centre of the starting square.



For questions 1–3, suppose the beetles start out at $(0,0)$, $(10,0)$, $(10,10)$, and $(0,10)$. The beetle starting at $(0,0)$ then crawls towards the one starting at $(10,0)$, which crawls towards the one starting at $(10,10)$, which crawls towards the one starting at $(0,10)$, which crawls towards the one starting at $(0,0)$.

1. If (x, y) is the position of the beetle that started at $(0,0)$ at some instant, what is the position of the beetle that started at $(10,0)$ at the same instant? [2]
2. Suppose (x, y) gives the position of the beetle that started at $(0,0)$. Use the fact that the beetle starting at $(0,0)$ crawls directly towards the one starting at $(10,0)$ at every instant to find an equation combining x , y , and $\frac{dy}{dx}$ that is satisfied by the curve followed by the beetle starting at $(0,0)$. [4]
3. Use Maple to solve the equation you obtained in answering 2. (Don't forget that the beetle starts at $(0,0)$ for the most complete answer.) [4]

Hint: The `dsolve` command may come in handy here...

Bonus. Without using any of your work in 1–3, determine how long it takes for all four beetles to meet after the start if they crawl at a constant rate of 1 *cm*. [2]

Hint: At any given instant each beetle is moving at right angles to the one it is pursuing and the one it is pursued by.