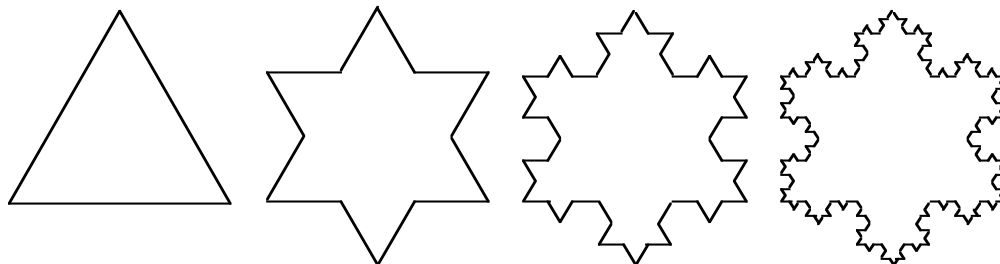


Mathematics 1101Y – Calculus I: Functions and calculus of one variable
TRENT UNIVERSITY, 2011–2012

Assignment #6
The snowflake curve*

Due on Thursday, 5 April, 2012.

Suppose one starts with an equilateral triangle with sides of length 1. If one modifies each of the line segments composing the triangle by cutting out the middle third of the segment, and then inserting an outward-pointing “tooth,” both of whose sides are as long as the removed third, one gets a six-pointed star. Repeating this for each of the line segments making up the star, then to each of the line segments making up the resulting figure, and so on, leads gives increasingly intricate curves with more and more corners. The first stages of the process are pictured in the diagram below:



Note that the lengths of the line segments at each stage are a third of the length of the segments at the preceding stage. For the sake of being definite, let’s say we have the triangle at step 0 of the process, the six-pointed star at step 1 of the process, the next shape at step 2 of the process, and so on. The curve which is the limit of this process, if one takes infinitely many steps, is often called the *snowflake curve**. We will investigate the length of this curve and the area of the region that it encloses.

1. What are the lengths of the curves we have at steps 0, 1, 2, and 3 of the process? [1]
2. What is the length L_n of the curve at step n of the process? [2]
3. Compute $\lim_{n \rightarrow \infty} L_n$ both by hand and with **Maple** to find the length of the snowflake curve. [2]
4. What are the areas of the regions enclosed by the curves we have at steps 0, 1, 2, and 3 of the process? [1]
5. What is the area A_n of the region enclosed by the curve at step n of the process? [2]
6. Compute $\lim_{n \rightarrow \infty} A_n$ both by hand and with **Maple** to find the area of the region enclosed by the snowflake curve. [2]

* Also known as the *Koch curve*. Note that “curve” does not imply smoothness here: although it is continuous, the snowflake curve has infinitely many corners and is therefore none too differentiable ...