

Mathematics 1101Y – Calculus I: Functions and calculus of one variable
TRENT UNIVERSITY, 2011–2012

Assignment #3

My name is Blond, Thames Blond.*

Due on Wednesday, 7 December, 2011.

Thames Blond, playboy heir to the Pale River Ale fortune and not-so-secret agent of MI7, is cruising along one of Saskatchewan's famously straight roads in his BMW at its top speed of 200 km/h , approaching the point where one of Saskatchewan's famously straight railroads crosses the road at a right angle. The last car of a train is passing the crossing at a speed of 100 km/h just as Blond is 1 km away. At this instant, Blond spots the infamous Dr. Yes looking out the back of that last train car. He immediately swerves to follow[†], keeping his BMW headed towards the last car of the train until he catches up.

1. If the train and Blond maintain their speeds of 100 km/h and 200 km/h , how far from the crossing does Blond catch up with the train? [10]

Hint: Supposing that the road lies along the x -axis and the railroad track along the y -axis, show that the BMW's path is the graph of a function satisfying the differential equation $2x \frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$. Solve this equation (Maple's `dsolve` command for solving differential equations may come in handy), assuming that $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 1$, and take it from there ...

* Apologies in advance to Ian Fleming and a certain prairie province.

† Remember that Saskatchewan is famously flat ...