**Mathematics 1101Y – Calculus I: functions and calculus of one variable**
TRENT UNIVERSITY, 2010–2011
**Final Examination**

**Time:** *14:00–17:00, on Tuesday, 26 April, 2011.* *Brought to you by* Стефан Біланюк.

**Instructions:** Do parts **X** and **Y** and, if you wish, part **Z**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

**Aids:** Calculator; one aid sheet (all sides!); one brain ($10^{100}$ neuron limit).

**Part X.** Do all three (3) of **1–3**.

**1.** Compute $\frac{dy}{dx}$ as best you can in any *three* (3) of **a–f**. *[15 = 3 × 5 each]*

**a.** $y = \cos(e^x)$ **b.** $y = \int_1^x e^t \ln(t)\, dt$ **c.** $y = x\ln(x)$

**d.** $y = \frac{\ln(x)}{x}$ **e.** $\arctan(x+y) = 0$ **f.** $\begin{aligned} x &= e^t \\ y &= e^{2t} \end{aligned}$

**2.** Evaluate any *three* (3) of the integrals **a–f**. *[15 = 3 × 5 each]*

**a.** $\int \frac{\ln(x)}{x}\, dx$ **b.** $\int \frac{1}{\sqrt{z^2-1}}\, dz$ **c.** $\int_1^4 \sqrt{x}\, dx$

**d.** $\int_0^1 \frac{x^2+2}{x^2+1}\, dx$ **e.** $\int \tan^2(w)\, dw$ **f.** $\int_0^{\ln(2)} e^{2t}\, dt$

**3.** Do any *five* (5) of **a–i**. *[25 = 5 × 5 each]*

**a.** Use the limit definition of the derivative to compute $g'(0)$ for $g(x) = 2x+1$.

**b.** Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ converges or diverges.

**c.** Find the Taylor series of $f(x) = e^{x+1}$ at $a = 0$.

**d.** Sketch the polar curve $r = 1$, where $0 \le \theta \le 2\pi$, and find the area of the region it encloses.

**e.** Sketch the surface obtained by rotating $y = \frac{x^2}{2}$, $0 \le x \le 2$, about the $y$-axis, and find its area.

**f.** Use the Right-hand Rule to compute the definite integral $\int_0^1 4x\, dx$.

**g.** Use the $\varepsilon - \delta$ definition of limits to verify that $\lim_{x\to 0}(2x-1) = -1$.

**h.** Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$.

**i.** Sketch the solid obtained by rotating the region bounded by $y = x$, $y = 0$, and $x = 2$, about the $x$-axis, and find its volume.

**Part Y.** Do any *three* (3) of **4–7**. *[45 = 3 × 15 each]*

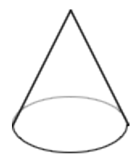
**4.** A zombie is dropped into a still pool, creating a circular ripple that moves outward from the point of impact at a constant speed. After 2 $s$ the length of the ripple is increasing at a rate of $2\pi$ $m/s$. How is the area enclosed by the ripple changing at this instant?

*Hint:* You have (just!) enough information to work out how the radius of the ripple changes with time.

**5.** Find all the intercepts, maximum, minimum, and inflection points, and all the vertical and horizontal asymptotes of $h(x) = \dfrac{x}{1-x^2}$, and sketch its graph.

**6.** Show that a cone with base radius 1 and height 2 has volume $\frac{2}{3}\pi$.

*Hint:* It's a solid of revolution ...



**7.** Do all *four* (4) of **a**–**d**.

- **a.** Use Taylor's formula to find the Taylor series of $f(x) = \sin(x)$ at $a = 0$. *[7]*
- **b.** Determine the radius and interval of convergence of this Taylor series. *[4]*
- **c.** Find the Taylor series of $g(x) = x\sin(x)$ at $a = 0$ by multiplying the Taylor series for $f(x) = \sin(x)$ by $x$. *[1]*
- **d.** Use Taylor's formula and your series from **c** to compute $g^{(16)}(0)$. *[3]*

*[Total = 100]*

**Part Z.** Bonus problems! Do them (or not), if you feel like it.

$\ln\left(\frac{1}{e}\right)$. Does $\displaystyle\lim_{n\to\infty}\left[\left(\sum_{k=1}^{n}\frac{1}{k}\right)-\ln(n)\right]$ exist? Explain why or why not. *[2]*

$\ln\left(\frac{1}{1}\right)$. Write a haiku touching on calculus or mathematics in general. *[2]*

**haiku?**

seventeen in three:
five and seven and five of
syllables in lines

I hope that you enjoyed this course,
and even learned a thing or two. :-)
Have a great summer!