Mathematics 1101Y – Calculus I: functions and calculus of one variable
TRENT UNIVERSITY, 2010–2011

Assignment #12
Tricks and treats with series
Due on Friday, 8 April, 2011.

1. Suppose you know that $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots = \frac{\pi^2}{8}$. Show that it follows that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{25} + \cdots = \frac{\pi^2}{6}$. *[3]*

NOTE: The given sums of the two series in **1** are correct.

2. Suppose x is a variable and $\displaystyle\sum_{n=0}^{\infty} a_n x^n = \left(\sum_{n=0}^{\infty} x^n\right)^2 = \left(1 + x + x^2 + x^3 + \cdots\right)^2$. Find a formula for a_n in terms of n. *[3]*

HINT: Work out the first few a_ns and look for a pattern.

3. Using the fact that $\displaystyle\arctan(x) = \int_0^x \frac{1}{1+t^2}\,dt$, find a formula for a_n in terms of n such that $\displaystyle\arctan(x) = \sum_{n=0}^{\infty} a_n x^n$. *[3]*

HINT: Express $\displaystyle\frac{1}{1+t^2}$ as a power series and then integrate it term-by-term.

4. Use your answer to **3** to find a series $\displaystyle\sum_{n=0}^{\infty} a_n$ whose sum is $\displaystyle\frac{\pi}{4}$. *[1]*

NOTE: Variations on this series have been used in the past to compute approximations to π. Nowadays people usually do so with other series that converge faster.