**Use the** $ϵ-δ$ **definition of limits** to verify that $\lim\_{x\to 2}(2x-3)=1$

We need to check that, given any $ϵ>0$ there is some $δ>0$ such that if$ 0<\left|x-2\right|<δ$, then $\left|\left(2x-3\right)-1\right|<ϵ.$

Given an $ϵ$ we reverse engineer the $δ$.

$\left|\left(2x-3\right)-1\right|<ϵ$

<=> $\left|2x-4\right|<ϵ$

<=> $2\left|x-2\right|<ϵ$

<=> $\left|x-2\right|<\frac{ϵ}{2}$

So $δ=\frac{ϵ}{2}$ does the job: if $\left|x-2\right|<δ=\frac{ϵ}{2}$ we can trace the equivalents backward to get

$\left|\left(2x-3\right)-1\right|<ϵ$.

**Graphing**

Find the domain, intercepts maxima and minima, and vertical and horizontal asymptotes of

$f\left(x\right)=\frac{e^{x}}{1+x^{2}}$ and sketch its graph.

Domain: $\left(-\infty , \infty \right)$ because $e^{x}and 1+x^{2}$ make snese for all $x$ and $1+x^{2}>0$

x intercept: None because $e^{x}$ is never 0

y intercept: $f\left(0\right)=\frac{e^{0}}{1+x^{2}}=\frac{1}{1}=1$

Domain has no end points so we only have to check for critical points.

$f^{'}\left(x\right)=\frac{e^{x}\left(1+x^{2}\right)×e^{x}(2x)}{\left(1+x^{2}\right)^{2}} $

$=\frac{e^{x}(x^{2}-2x+1)}{\left(1+x^{2}\right)^{2}}=\frac{e^{x}\left(x-1\right)^{2}}{\left(1+x^{2}\right)^{2}}$

Critical Points

$f^{'}\left(x\right)= 0$ when

$x=1$

$f^{'}\left(x\right)$ is undefined when

Denominator is never equal to zero

|  |  |  |  |
| --- | --- | --- | --- |
| x | $$\left(-\infty , 1\right)$$ | 1 | $$(1, \infty )$$ |
| $$f^{'}\left(x\right)$$ | + | 0 | + |
| $$f(x)$$ | increasing | neither max nor min | increasing |

Vertical Asymptotes: None. $f\left(x\right)$ is defined & continuous everywhere

|  |  |
| --- | --- |
| Horizontal Asymptotes: $\lim\_{x\to +\infty }f\left(x\right)=\lim\_{x\to +\infty }\frac{e^{x}}{1+x^{2}}\begin{matrix}\rightarrow \infty \\\rightarrow \infty \end{matrix}$Using l’Hopital’s Rule: $\lim\_{x\to +\infty }\frac{e^{x}}{2x}=\begin{matrix}\rightarrow \infty \\\rightarrow \infty \end{matrix}$Using l’Hopital’s Rule: $\lim\_{x\to +\infty }\frac{e^{x}}{2}=\infty $ | $$\lim\_{x\to -\infty }f\left(x\right)=\lim\_{x\to -\infty }\frac{e^{x}}{1+x^{2}}\begin{matrix}\rightarrow 0\\\rightarrow \infty \end{matrix}=0$$So we have a horizontal asymptote of $y=0$ in the negative direction and none in the positive |