Math 1100 — Calculus, Quiz #1A — 2009-09-21

Define the functions $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ by $f(x) = 3\,x$ and $g(x) = x+1$ for all $x \in \mathbb{R}$.

1. Find a formula for each of the following functions:

(10)
    (a) $f \circ f$.
    Solution: $f \circ f(x) = 3 \cdot 3 \cdot x = \boxed{9\,x,}$ for all $x \in \mathbb{R}$. $\square$

(10)
    (b) $f \circ f \circ f$.
    Solution: $f \circ f \circ f(x) = 3 \cdot 3 \cdot 3 \cdot x = \boxed{27\,x,}$ for all $x \in \mathbb{R}$. $\square$

(10)
    (c) $g \circ g$.
    Solution: $g \circ g(x) = x + 1 + 1 = \boxed{x + 2,}$ for all $x \in \mathbb{R}$. $\square$

(10)
    (d) $g \circ g \circ g$.
    Solution: $g \circ g \circ g(x) = x + 1 + 1 + 1 = \boxed{x + 3,}$ for all $x \in \mathbb{R}$. $\square$

(10)
    (e) $f \circ g$.
    Solution: $f \circ g(x) = f[g(x)] = f(x + 1) = 3(x + 1) = \boxed{3x + 3,}$ for all $x \in \mathbb{R}$. $\square$

(10)
    (f) $g \circ f$.
    Solution: $g \circ f(x) = g[f(x)] = g(3\,x) = \boxed{3\,x + 1,}$ for all $x \in \mathbb{R}$. $\square$

2. Define the function $f : (-\infty, 9] \longrightarrow \mathbb{R}_+$ by $f(x) = \sqrt{27 - 3x}$ for all $x \in (-\infty, 9]$.

(10)
    (a) Why is $f(x)$ not well-defined (i.e. not a real number) if $x > 9$?
    Solution: If $x > 9$, then $3x > 27$, so $27 - 3x < 0$, so $\sqrt{27 - 3x}$ is not well-defined (as a real number). $\square$

(20)
    (b) Find a formula for the function f^{-1}.
    Solution: For any $x \in (-\infty, 9]$ and $y \in \mathbb{R}_+$, we have:

$$\left(y = f(x)\right) \iff \left(y = \sqrt{27 - 3x}\right) \iff \left(y^2 = 27 - 3x\right) \iff \left(y^2/3 = 9 - x\right)$$
$$\iff \left(x = 9 - y^2/3\right) \iff \left(f^{-1}(y) = 9 - y^2/3\right)$$

    We conclude that $\boxed{f^{-1}(y) = 9 - y^2/3}$ for all $y \in \mathbb{R}_+$. $\square$

(10)
    (c) What is the range of the function f^{-1}?
    Solution: The range of f^{-1} is the domain of f —that is, the interval $\boxed{(-\infty, 9]}$. $\square$