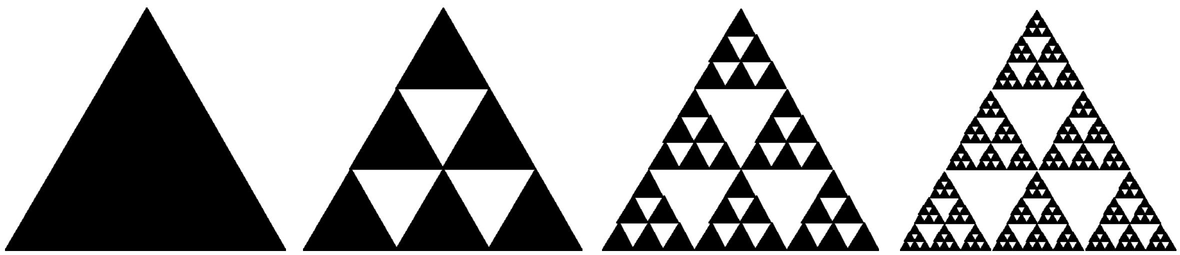
Mathematics 110 – Calculus of one variable
Trent University 2003-2004

ASSIGNMENT #1
Due: Wednesday, 1 October, 2003

Fractal nonsense or nonsense fractal?

Start with an equilateral triangle of area one, set up so its base is horizontal. At step one, divide it up into nine equal equilateral subtriangles and remove the (insides of) three that point downwards. At step two, do the same to each of the six surviving subtriangles. At step three, do the same to each of the thirty six surviving subsubtriangles. Here's a picture:



Now just keep on going! The basic problem is to figure out what the shape that is left after infinitely many steps is like. (Something *is* left over. For example, the corners of the original triangle, of the subtriangles, of the subsubtriangles, *etc.*, never get removed.) What one has to work with is understanding the process and the fact that the process generates ever closer approximations of the "final" shape.

1. What is the area of the final shape? *[2]*

2. What is the length of the border of the final shape? *[2]*
[The borders inside the shape do count!]

3. Considering your answers to **1** and **2**, can the shape in question be real? Why or why not? *[1]*

Limitations

It's pretty obvious that $\lim\limits_{t \to +\infty} \frac{1}{t} = 0$ and that $\lim\limits_{u \to 0^+} \frac{1}{u} = +\infty$. This relationship, together with the following fact,

4. Suppose f is some function whose domain includes the interval $(0, c)$, where c is some constant greater than 0. Use the $\varepsilon - \delta$ and $\varepsilon - N$ definitions of limits to show that $\lim\limits_{t \to +\infty} f(t) = \lim\limits_{u \to 0^+} f\left(\frac{1}{u}\right)$. *[3]*

... can come in handy occasionally when faced with some otherwise messy limits.

5. Use **4** to explain why $\lim\limits_{x \to 0^+} \sin\left(\frac{1}{x}\right)$ does not exist. *[2]*

[Total = 10]

Bonus!

2π. Suppose h is a function such that for *every* sequence a_n with $\lim\limits_{n \to \infty} a_n = a$, it is true that $\lim\limits_{n \to \infty} h(a_n) = h(a)$. Does h have to be continuous at a? Prove it does or find a counterexample. *[2]*