

Mathematics 4790H – Analysis II: Topology and Measure
TRENT UNIVERSITY, Winter 2025

Instructor

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Department of Mathematics

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Prerequisite: MATH 3790H. Not open to students with credit for MATH 3700H.

Texts

For the metric spaces part of the course, we will use *An Introduction to Metric Spaces*, by S.E. Andersson, A. Björn, and D. Wiman, which may be downloaded from:

https://courses.mai.liu.se/GU/TATA34/An_Introduction_to_Metric_Spaces.pdf

For the measure theory part of the course, we will use *Measure, Integration & Real Analysis*, by Sheldon Axler, which may be downloaded from:

<https://doi.org/10.1007/978-3-030-33143-6>

Copies of both texts will also be available from the course Blackboard site.

Meetings: Tuesdays 16:00-17:50 in CCW C3 and Fridays 13:00-14:50 in ENW 112. Please consult the Academic Timetable at www.trentu.ca/registrar/timetable/undergraduate for up-to-date times and locations. An attempt will be made to record the lectures for those who miss them.

Marking scheme

There will be at least eleven weekly assignments and a take-home final examination. Please consult the schedule below for due dates. The work will weigh as follows:

Best 10 assignments (6.5% each) 65%

Take-home final examination 35%

At least 25% of the course marks will be obtained by the final date (Friday, 7 March) to withdraw from Winter courses. Students who are unable to submit an assignment or the take-home final exam on the due date, or who miss more than one assignment, for reasons beyond their control should contact the instructor as soon as possible.

This scheme may also be modified for individual students in exceptional circumstances, such as a lengthy absence due to illness. Any such modification will require the agreement of both the student and the instructor.

Content & Learning Outcomes

MATH 4790H is an introduction to the topology of metric spaces and to measure theory on the real line. Successful students will acquire experience with using the basic definitions and results about metric spaces, including open and closed sets, sequences and their limits, completeness, and continuity. Various examples will be studied, including normed linear spaces, sequential and function spaces, with particular attention to the real line. Students will also get experience with the definitions and basic results about Lebesgue measure on the real line, the Lebesgue integral, and some applications of these ideas to function spaces.

Schedule

Please note that where the material covered is concerned, the schedule below is a polite fiction: no lesson plan survives contact with students unchanged. All the more so here, since this is the very first iteration of MATH 4790H ...

Week 1. (6-10 January) Classes begin on Monday, 6 January. Metric spaces, various examples of metric spaces, including \mathbb{R} , normed linear spaces, sequence and function spaces.

Week 2. (13-17 January) Open and closed sets, sequences and convergence, Cauchy sequences, completeness. Assignment #1 due on Friday, 17 January.

Week 3. (20-24 January) Continuous functions, compactness. Assignment #2 due on Friday, 24 January.

Week 4. (27-31 January) Outer measure on \mathbb{R} , the good and the bad. Assignment #3 due on Friday, 31 January.

Week 5. (3-7 February) σ -algebras, measurable spaces and sets, Borel subsets of \mathbb{R} . Assignment #4 due on Friday, 7 February.

Week 6. (10-14 February) Measurable functions, measures and their properties. Assignment #5 due on Friday, 14 February.

Reading Week. (17-21 February) University closed on Family Day, Monday, 17 February.

Week 7. (24-28 February) Lebesgue measure, sets of measure 0, Lebesgue measurable sets. Assignment #6 due on Friday, 28 February.

Week 8. (3-7 March) Pointwise and uniform convergence of measurable functions. Assignment #7 on Friday, 7 March. *The last date to withdraw from Winter courses is Friday, 7 March.*

Week 9. (10-14 March) Egorov's and Luzin's Theorems. Approximation of measurable functions by simple functions. Assignment #8 due on Friday, 14 March.

Week 10. (17-21 March) Lebesgue measurable functions, integration with respect to Lebesgue measure. Assignment #9 due on Friday, 21 March.

Week 11. (24-28 March) Monotone Convergence Theorem, properties of the Lebesgue integral. Assignment #10 due and take-home final examination distributed on Friday, 28 March.

Week 12. (31 March – 4 April) Bounded and Dominated Convergence Theorems. Assignment #11 due on Friday, 4 April. *Friday, 4 April, is the last day of classes.*

Winter Examination Period. (7-23 April) Take-home final examination due on Thursday, 17 April. *University closed on Good Friday, 18 April.*

Academic Integrity

Academic dishonesty, which includes plagiarism and cheating, is an extremely serious academic offence and carries penalties varying from a 0 grade on an assignment to expulsion from the University. Definitions, penalties, and procedures for dealing with plagiarism and cheating are set out in Trent University's Academic Integrity Policy. You have a responsibility to educate yourself – unfamiliarity with the policy is not an excuse. You are strongly encouraged to visit Trent's Academic Integrity website to learn more – www.trentu.ca/academicintegrity

For clarity, the following guidelines will apply in MATH 4790H:

You are permitted and encouraged to work together and ask anyone willing (especially the instructor!) for explanations, hints, and suggestions on the assignments, and to use what software and consult whatever sources you wish. However, **all work submitted for credit must be written up entirely by yourself, giving due credit to all relevant sources of help and information.** The restrictions applicable to the take-home final exam will be spelled out on the exam.

Access to Instruction

It is Trent University's intent to create an inclusive learning environment. If a student has a disability and documentation from a regulated health care practitioner and feels that they may need accommodations to succeed in a course, the student should contact the Student Accessibility Services Office (SAS) at the respective campus as soon as possible.

Last modified 2024-11-24.