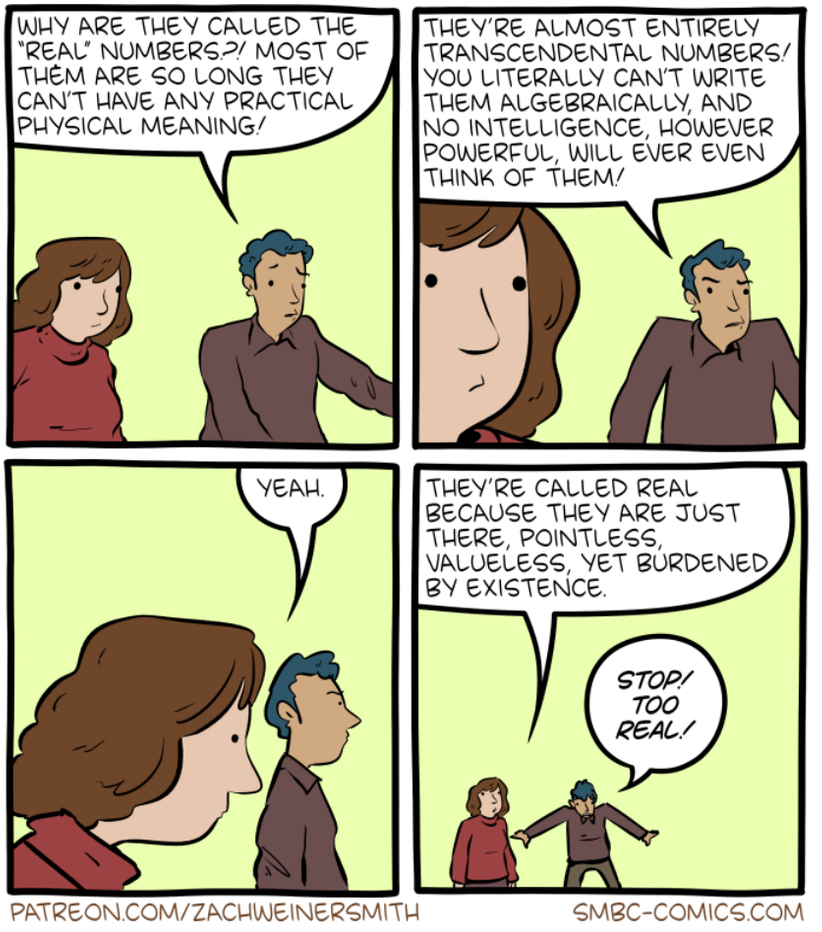
Mathematics 4790H – Analysis II: Topology and Measure
Trent University, Winter 2025

Assignment #9
Convergence and Continuity
Due on Friday, 21 March.[*]

1. Suppose $[a, b] \subset \mathbb{R}$, where $a, b \in \mathbb{R}$, and $\{f_k\}$ is a sequence of continuous functions $f_k : [a, b] \to \mathbb{R}$, with $f_k \to f$ for some function $f : [a, b] \to \mathbb{R}$, such that for each $x \in [a, b]$, we have $f_0(x) \leq f_1(x) \leq f_2(x) \leq \cdots \leq f(x) < \infty$. Show that f is continuous on $[a, b]$ if and only if $f_k \xrightarrow{\text{unif}} f$ uniformly on $[a, b]$. *[10]*

Hint: One direction is immediate from something we did in class ...



[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.