Mathematics 4790H – Analysis II: Topology and Measure

Trent University, Winter 2025

Assignment #8
Borel and Lebesgue Measurable Sets

Due on Friday, 14 March.[*]

1. Show that there exists a collection of closed intervals of $\mathbb{R}$ whose union is not a Borel set. *[5]*

2. Show that if $A \subseteq \mathbb{R}$ is Lebesgue measurable and $r \in \mathbb{R}$, then $rA = \{\, ra \mid a \in A \,\}$ is also Lebesgue measurable. *[5]*

Love and Tensor Algebra

Come, let us hasten to a higher plane,
Where dyads tread the fairy fields of Venn,
Their indices bedecked from one to n,
Commingled in an endless Markov chain!

Come, every frustrum longs to be a cone,
And every vector dreams of matrices.
Hark to the gentle gradient of the breeze:
It whispers of a more ergodic zone.

In Riemann, Hilbert or in Banach space
Let superscripts and subscripts go their ways.
Our asymptotes no longer out of phase,
We shall encounter, counting, face to face.

I'll grant thee random access to my heart,
Thou'lt tell me all the constants of thy love;
And so we two shall all love's lemmas prove,
And in our bound partition never part.

For what did Cauchy know, or Christoffel,
Or Fourier, or any Boole or Euler,
Wielding their compasses, their pens and rulers,
Of thy supernal sinusoidal spell?

Cancel me not for what shall then remain?
Abscissas, some mantissas, modules, modes,
A root or two, a torus and a node:
The inverse of my verse, a null domain.

Ellipse of bliss, converge, O lips divine!
The product of our scalars is defined!
Cyberiad draws nigh, and the skew mind
Cuts capers like a happy haversine.

I see the eigenvalue in thine eye,
I hear the tender tensor in thy sigh.
Bernoulli would have been content to die,
Had he but known such $a^2 cos(2\phi)$!

From *The Cyberiad* by Stanislaw Lem (translated from Polish by Michael Kandel).

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.