Mathematics 4790H – Analysis II: Topology and Measure
TRENT UNIVERSITY, Winter 2025

Assignment #5
Continuity and Borel Sets
Due on Friday, 14 February.[*]

The following questions are taken from the textbook, *Measure, Integration, & Real Analysis*; see Exercises 2B, #12, p. 38.

In what follows, suppose $f : \mathbb{R} \to \mathbb{R}$ is an arbitrary function.

1. For each integer $k > 0$ let

$$G_k = \left\{ a \in \mathbb{R} \mid \exists \delta > 0 \, \forall b, \, c \in (a - \delta, a + \delta) : |f(b) - f(c)| < \frac{1}{k} \right\}.$$

Show that G_k is an open subset of $\mathbb{R}$ for every $k > 0$. *[4]*

2. Show that the set of points at which f is continuous is $\bigcap_{k=1}^{\infty} G_k$. *[5]*

3. Show that the set of points at which f is continuous is a Borel set. *[1]*

A would-be Abelian group
Was trying its best to commute;
But it found the GO train
An unbearable pain,
So it rolled into town as a loop!

An abstract algebra limerick by Nicole Bauberger, from 1992.
Nicole was a Math & English major at Trent who became a painter.

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.