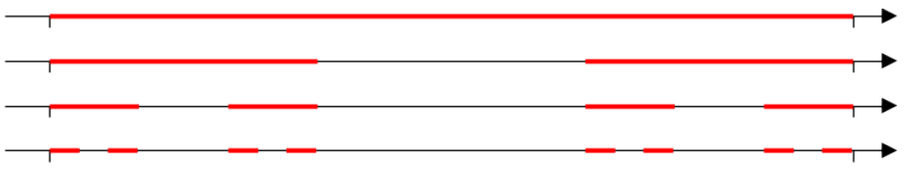
Mathematics 4790H – Analysis II: Topology and Measure
TRENT UNIVERSITY, Winter 2025

Assignment #4
The Cantor Set
Due on Friday, 7 February.[*]

The Cantor set is defined by the following process:

Step 0. Start with the closed unit interval $[0, 1]$.
Step 1. Remove the open middle third of the interval, leaving $\left[0, \frac{1}{3}\right]$ and $\left[\frac{2}{3}, 1\right]$.
Step 2. Remove the open middle third of each of the remaining intervals, leaving $\left[0, \frac{1}{9}\right]$, $\left[\frac{2}{9}, \frac{3}{9}\right]$, $\left[\frac{6}{9}, \frac{7}{9}\right]$ and $\left[\frac{8}{9}, 1\right]$.
⋮
Step n. Remove the open middle third of each of the intervals remaining after step $n-1$, leaving $\left[0, \frac{1}{3^n}\right]$, $\left[\frac{2}{3^n}, \frac{3}{3^n}\right]$, $\ldots$, $\left[\frac{3^n-1}{3^n}, 1\right]$
⋮

Here is a picture of Steps 0 through 3:



The Cantor set is the limit of the process, *i.e.* what remains after infinitely many steps.

1. Explain why the Cantor set is uncountable. *[2]*

2. Prove that the Cantor set is compact. *[4]*

3. Show that the Cantor set has outer measure 0. *[4]*

Good Advice

Shun advice
at any price –
that's what I call
good advice.

Piet Hein

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.