

Mathematics 4790H – Analysis II: Topology and Measure

TRENT UNIVERSITY, Winter 2025

Assignment #3

Shrink

*Due on Friday, 31 January.**

Recall that a metric space is said to be *complete* if every Cauchy sequence in the metric space has a limit in the space.

Suppose (X, d) is a metric space. A function $f : X \rightarrow X$ is said to be a *contraction mapping* if for some $k \in [0, 1)$ and all $x, y \in X$, $d(f(x), f(y)) < k \cdot d(x, y)$.

1. Suppose (X, d) is a metric space and $f : X \rightarrow X$ is a contraction mapping. Show that f is uniformly continuous on X . [5]
2. Suppose (X, d) is a complete metric space and $f : X \rightarrow X$ is a contraction mapping. Show that f has a unique *fixed point*, i.e. an $x \in X$ such that $f(x) = x$. [5]

Mark Twain (Sam Clemens), from *Answers to Correspondents*:

”ARITHMETICUS” Virginia, Nevada. - ”If it would take a cannonball $3 \frac{1}{8}$ seconds to travel four miles, and $3 \frac{3}{8}$ seconds to travel the next four, and $3 \frac{5}{8}$ to travel the next four, and if its rate of progress continued to diminish in the same ratio, how long would it take to go fifteen hundred million miles?”

I don’t know.

A novel application of contraction mappings appears in Larry Niven’s widely reprinted fantasy short story *Convergent Series*, which originally appeared under the title *The Long Night* in the March 1967 issue of *The Magazine of Fantasy & Science Fiction*. Look it up! It is one of the two places I know of in his works, not counting some author’s notes, that Niven’s having been a math major is apparent, the other being a footnote in his essay *Man of Steel, Woman of Kleenex*. (A hilarious rigorous analysis of some of the inherent problems in Superman’s sex life.)

* Please submit your solutions, preferably as a single pdf, via Blackboard’s Assignments module. If that fails, please submit them to the instructor on paper or via email to sbilaniuk@trentu.ca as soon as you can.