Mathematics 4790H – Analysis II: Topology and Measure

Trent University, Winter 2025

Assignment #1

Cases of Completeness

Due on Friday, 24 January.[*]

Recall that a metric space is said to be *complete* if every Cauchy sequence in the metric space has a limit in the space.

$L^\infty([0,1])$ is the metric space whose points are all the continuous functions $f : [0,1] \to \mathbb{R}$, equipped with the metric $d(f,g) = \sup\{\,|f(x) - g(x)|\; 0 \le x \le 1\,\}$, sometimes written as $d(f,g) = \max\{\,|f(x) - g(x)|\; 0 \le x \le 1\,\}$. On the other hand, $L^1([0,1])$ is the metric space which has the same points, *i.e.* the continuous functions $f : [0,1] \to \mathbb{R}$, but equipped with the metric $d(f,g) = \displaystyle\int_0^1 |f(x) - g(x)|\; dx$

1. Show that $L^\infty([0,1])$ is complete. *[5]*

2. Show that $L^1([0,1])$ is not complete. *[5]*

JENNET: In the pursuit of alchemy.
In refusing to accept the dictum "It is
What it is." Poor father. In the end he walked
In Science like the densest night. And yet
He was greatly gifted.
When he was born he gave an algebraic
Cry; at one glance measured the cubic content
Of that ivory cone his mother's breast
And multiplied his appetite by five.
So he matured by a progression, gained
Experience by correlation, expanded
Into marriage by contraction, and by
Certain physical dynamics
Formulated me. And on he went
Still deeper into the calculating twilight
Under the twinkling of five-pointed figures
Till Truth became the sum of sums
And Death the long division. My poor father.
What years and powers he wasted.
He thought he could change the matter of the world
From the poles to the simultaneous equator
By strange experiment and by describing
Numerical parabolas.

From *The Lady's Not for Burning* by Christopher Fry.

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can.