

**Mathematics 4790H – Analysis II: Topology and Measure**

TRENT UNIVERSITY, Winter 2025

**Assignment #10**

**Basic Bounds and Cute (?) Convergence**

*Due on Friday, 28 March.\**

1. Suppose  $f : \mathbb{R} \rightarrow \mathbb{R}$  is Lebesgue-measurable and bounded, and  $E \subset \mathbb{R}$  is a Lebesgue-measurable set with  $|E| < \infty$ . Verify that

$$|E| \cdot \inf \{ f(x) \mid x \in E \} \leq \int_E f(x) dx \leq |E| \cdot \sup \{ f(x) \mid x \in E \}. \quad [4]$$

2. Give an example of a sequence  $\{f_i\}$  of simple and Lebesgue-measurable functions  $f_i : \mathbb{R} \rightarrow [0, \infty)$  such that  $\lim_{i \rightarrow \infty} f_i(x) = 0$  for all  $x \in \mathbb{R}$ , but  $\lim_{i \rightarrow \infty} \int_{\mathbb{R}} f_i(x) dx = 1$ . [6]

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\* Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to [sbilaniuk@trentu.ca](mailto:sbilaniuk@trentu.ca) as soon as you can.