## Mathematics 4790H – Analysis II: Topology and Measure TRENT UNIVERSITY, Winter 2025

## Assignment #1 Playing With Sequences Due on Friday, 17 January.\*

Suppose  $[a_n^k]$  is an infinite matrix of real numbers such that for all  $n \ge 0$ ,  $\lim_{k \to \infty} a_n^k = a_n$  for some real number  $a_n$ , and for all  $k \ge 0$ ,  $\lim_{n \to \infty} a_n^k = a^k$  for some real number  $a^k$ .

$a_{0}^{0}$	$a_1^0$	$a_{2}^{0}$	$a_{3}^{0}$	$a_{4}^{0}$	•••	$\rightarrow$	$a^0$
$a_{0}^{1}$	$a_1^1$	$a_2^1$	$a_3^1$	$a_4^1$	•••	$\rightarrow$	$a^1$
$a_0^2$	$a_{1}^{2}$	$a_{2}^{2}$	$a_{3}^{2}$	$a_4^2$	•••	$\rightarrow$	$a^2$
$a_0^3$	$a_{1}^{3}$	$a_{2}^{3}$	$a_{3}^{3}$	$a_{4}^{3}$	•••	$\rightarrow$	$a^3$
$a_0^4$	$a_1^4$	$a_{2}^{4}$	$a_3^4$	$a_4^4$	•••	$\rightarrow$	$a^4$
÷	:	:	÷	:	·		:
I	I	I	I	I	•		I
$\downarrow$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$			$\downarrow$
$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	•••	$\rightarrow$	?

- 1. Give an example to show that even if  $\lim_{n\to\infty} a_n$  and  $\lim_{k\to\infty} a^k$  both exist, they need not be equal. [5]
- 2. Give a condition on the limits involved in this setup that ensures that  $\lim_{n \to \infty} a_n = \lim_{k \to \infty} a^k$  if both limits exist, and prove that it does. [5]

$$\int_{-\infty}^{\infty} death^{-x^{2}} dx = \left[ \int_{-\infty}^{\infty} comes \ after^{-x^{2}} dx \int_{-\infty}^{\infty} a \ breath^{-y^{2}} dy \right]^{1/2}$$
$$= \left[ \int_{0}^{2\pi} \int_{0}^{\infty} e^{-r^{2}} therefore \ \theta \right]^{1/2}$$
$$= \left[ \pi \int_{0}^{\infty} breathe \ slowly^{-u} du \right]^{1/2}$$
$$= \sqrt{\pi}$$

Radoslav Rochallyi, "where the parched scythe of fate begs for mercy." # mathaeata, 2021, ISBN: 978-8097373719.

<sup>\*</sup> Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to sbilaniuk@trentu.ca as soon as you can.