Mathematics-Computer Science 4215H – Mathematical Logic
TRENT UNIVERSITY, Winter 2021

Take-home Final Examination
Due on Wednesday, 21 April.

Instructions: All references below are to the textbook[0]. You may consult whatever sources you wish, so long as you acknowledge those you actually use. However, you may not consult other people, except to ask the instructor to clarify problems. Do all three of Parts I–III and, if you wish, Part IV as well.

Part I. Do any *two* (2) of **1**–**4**. *[20 = 2 × 10 each]*

1. Theorem 1.12

2. Proposition 2.9

3. Problem 3.9 (7)

4. Theorem 4.7

Part II. Do any *two* (2) of **5**–**8**. *[20 = 2 × 10 each]*

5. Suppose $\mathcal{L}$ is the first-order language whose only non-logical symbol is the 3-place function symbol f. Determine the possible lengths of terms of $\mathcal{L}$.

6. Proposition 6.16, parts (1), (2), and (6)

7. Proposition 7.4 (for axiom schema (A4) and (A7) only)

8. Is it true that if Σ is a set of sentences of a first-order language $\mathcal{L}$, then Σ is maximally-consistent if and only if $\mathrm{Th}(\Sigma) = \Sigma$? Prove it or give a counterexample.

Part III. Do any *two* (2) of **9**–**12**. *[30 = 2 × 15 each]*

9. Prove the Interpolation Theorem for propositional logic:

> *Suppose α and β are formulas of $\mathcal{L}_P$ such that $\vdash \alpha \to \beta$. Then there is a formula γ of $\mathcal{L}_P$ such that both $\vdash \alpha \to \gamma$ and $\vdash \gamma \to \beta$, and every atomic formula that appears in γ either appears in both or neither of α and β.*

10. Problem 9.1(2)

11. Theorem 9.2 [Ramsey's Theorem], using first-order logic rather than propositional logic. You may assume Lemma 9.3 [Infinite Ramsey's Theorem].

12. Proposition 9.4

[Total = 70]

Part IV. Bonus!

∅. Write an original poem that is *not* a limerick about logic. *[1]*

ENJOY THE SUMMER!

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[0] *A Problem Course in Mathematical Logic*, Version 1.6.