Mathematics-Computer Science 4215H – Mathematical Logic
TRENT UNIVERSITY, Winter 2021
Assignment #9
Due on Friday, 26 March.

Do all of the following problems, all of which are straight out of the textbook[0] (which explains the numbering), reproduced here for your convenience.

6.12. *[Problem 6.12]* Show that $\forall y \, y = y$ is a tautology and that $\exists y \, \neg y = y$ is a contradiction. *[2]*

6.14. *[Problem 6.14]* Show that a set of formulas Σ is satisfiable if and only if there is no contradiction χ such that $\Sigma \vDash \chi$. *[2]*

6.15. *[Proposition 6.15]* Suppose $\mathfrak{M}$ is a structure for $\mathcal{L}$ and α and β are sentences of $\mathcal{L}$. Then:

    (1) $\mathfrak{M} \vDash \neg\alpha$ if and only if $\mathfrak{M} \nvDash \alpha$. *[2]*
    (2) $\mathfrak{M} \vDash \alpha \rightarrow \beta$ if and only if $\mathfrak{M} \vDash \alpha$ whenever $\mathfrak{M} \vDash \beta$. *[2]*
    (6) $\mathfrak{M} \vDash \forall x \, \alpha$ if and only if $\mathfrak{M} \vDash \alpha$. *[2]*

6.10. *[Proposition 6.10]* Suppose α and β are formulas of some first-order language. Then $\{\, (\alpha \rightarrow \beta),\, \alpha \,\} \vDash \beta$. *[2]*

7.1 *[Problem 7.1]* (1) Is x substitutable for z in ψ if ψ is $z = x \rightarrow \forall z \, z = x$? If so, what is ψ_x^z? *[1]*
    (2) Show that if t is any term and σ is a sentence, then t is substitutable in σ for any variable x. What is σ_t^x? *[2]*

[Total = 15]