

## Mathematics-Computer Science 4215H – Mathematical Logic

TRENT UNIVERSITY, Winter 2021

### Assignment #8

Due on Friday, 19 March.

Do all of the following problems, all of which are straight out of the textbook<sup>0</sup> (which explains the numbering), reproduced here for your convenience.

- 6.6.** [Problem 6.6]  $\mathfrak{Q} = (\mathbb{Q}, <)$  is a structure for  $\mathcal{L}_O$ . For each of the following formulas  $\varphi$  of  $\mathcal{L}_O$ , determine whether or not  $\mathfrak{Q} \models \varphi$ .
- (1)  $\forall v_0 \exists v_2 v_0 < v_2$  [1]
  - (3)  $\forall v_4 \forall v_5 \forall v_6 (v_4 < v_5 \rightarrow (v_5 < v_6 \rightarrow v_4 < v_6))$  [1]
- 6.7.** [Lemma 6.7] Suppose  $\mathfrak{M}$  is a structure for  $\mathcal{L}$ ,  $t$  is a term of  $\mathcal{L}$ , and  $r$  and  $s$  are assignments for  $\mathfrak{M}$  such that  $r(x) = s(x)$  for every variable  $x$  which occurs in  $t$ . Then  $\mathbf{r}(t) = \mathbf{s}(t)$ . [2]
- 6.8.** [Proposition 6.8] Suppose  $\mathfrak{M}$  is a structure for  $\mathcal{L}$ ,  $\varphi$  is a formula of  $\mathcal{L}$ , and  $r$  and  $s$  are assignments for  $\mathfrak{M}$  such that  $r(x) = s(x)$  for every variable  $x$  which occurs free in  $\varphi$ . Then  $\mathfrak{M} \models \varphi[r]$  if and only if  $\mathfrak{M} \models \varphi[s]$ . [4]
- 6.9.** [Corollary 6.9] Suppose  $\mathfrak{M}$  is a structure for  $\mathcal{L}$  and  $\sigma$  is a sentence of  $\mathcal{L}$ . Then  $\mathfrak{M} \models \sigma$  if and only if there is some assignment  $s : V \rightarrow |\mathfrak{M}|$  for  $\mathfrak{M}$  such that  $\mathfrak{M} \models \sigma[s]$ . [2]
- 6.10.** [Proposition 6.10] Suppose  $\alpha$  and  $\beta$  are formulas of some first-order language. Then  $\{(\alpha \rightarrow \beta), \alpha\} \models \beta$ . [3]
- 6.11.** [Proposition 6.11] Suppose  $\Sigma$  is a set of formulas and  $\psi$  and  $\rho$  are formulas of some first-order language. Then  $\Sigma \cup \{\psi\} \models \rho$  if and only if  $\Sigma \models (\psi \rightarrow \rho)$ . [2]

[Total = 15]

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<sup>0</sup> A Problem Course in Mathematical Logic, Version 1.6.