Mathematics-Computer Science 4215H – Mathematical Logic
TRENT UNIVERSITY, Winter 2021

Assignment #7
Due on Friday, 12 March.

Do all of the following problems, all of which are straight out of the textbook[0] (which explains the numbering), reproduced here for your convenience.

5.13. *[Problem 5.13]* Which of the languages given in Example 5.2 are extensions of other languages given in Example 5.2? *[2]*

5.14. *[Proposition 5.14]* Suppose $\mathcal{L}$ is a first-order language and $\mathcal{L}'$ is an extension of $\mathcal{L}$. Then every formula of $\mathcal{L}$ is a formula of $\mathcal{L}'$. *[3]*

6.1. *[Problem 6.1]* The first-order languages referred to below were all defined in Example 5.2.

(2) Determine whether $\mathfrak{Q} = (\mathbb{Q}, <)$ is a structure for each of $\mathcal{L}_=$, $\mathcal{L}_F$, and $\mathcal{L}_S$. *[1]*
(3) Give three different structures for $\mathcal{L}_F$ which are not fields. *[1]*

6.2. *[Problem 6.2]* $\mathfrak{N} = (\mathbb{N}, 0, S, +, \cdot, E)$ is a structure for $\mathcal{L}_N$. Let $s : V \to \mathbb{N}$ be the assignment defined by $s(v_k) = k{+}1$. What are $\mathbf{s}(E{+}v_{19}v_1{\cdot}0v_{45})$ and $\mathbf{s}(SSS{+}E0v_6v_7)$? *[1]*

6.3. *[Proposition 6.3]* $\mathbf{s}$ is unique, *i.e.* given an assignment s, no other function $T \to |\mathfrak{M}|$ satisfies conditions 1–3 in Definition 6.3. *[3]*

6.4. *[Problem 6.4]* Let $\mathfrak{N}$ be the structure for $\mathcal{L}_N$ in Problem 6.2. Let $p : V \to \mathbb{N}$ be defined by $p(v_{2k}) = k$ and $p(v_{2k+1}) = k$. Verify that:

(1) $\mathfrak{N} \vDash \forall w\,(\neg Sw = 0)\,[p]$ *[2]*
(2) $\mathfrak{N} \nvDash \forall x \exists y\, x + y = 0\,[p]$ *[2]*

[Total = 15]

[0] *A Problem Course in Mathematical Logic*, Version 1.6.