Mathematics-Computer Science 4215H – **Mathematical Logic**
TRENT UNIVERSITY, Winter 2021
Assignment #6
Due on Friday, 5 March.

Do all of the following problems, all of which are straight out of the textbook[0] (which explains the numbering), reproduced here for your convenience.

5.1. *[Problem 5.1]* Which of the following are terms of one of the languages defined in Examples 5.1 and 5.2? If so, which of these language(s) are they terms of; if not, why not?
(1) $\cdot v_2$ (3) $|1 + v_3 0$ (5) $+ + \cdot + 00000$ *[1.5 = 3×0.5 each]*

5.2. *[Problem 5.2]* Choose one of the languages defined in Examples 5.1 and 5.2 which has terms of length greater than one and determine the possible lengths of terms of this language. *[2]*

5.4. *[Problem 5.4]* Which of the following are formulas of one of the languages defined in Examples 5.1 and 5.2? If so, which of these language(s) are they formulas of; if not, why not?
(1) $= 0 + v_7 \cdot 1 v_3$ (3) $(|v_2 0 \rightarrow \cdot 01)$ (5) $< +01|v_1 v_3$ *[1.5 = 3×0.5 each]*

5.6. *[Problem 5.6]* Choose one of the languages defined in Examples 5.1 and 5.2 and determine the possible lengths of formulas of this language. [Do this for the language you chose in Problem 5.2.] *[2]*

5.8. *[Problem 5.8]* In each case, write down a formula of the given language expressing the given informal statement.
(2) "There is an empty set" in $\mathcal{L}_S$. *[1]*
(4) "$n^0 = 1$ for every n different from 0" in $\mathcal{L}_N$. *[1]*

5.9. *[Problem 5.9]* Define first-order languages to deal with the following structures and, in each case, an appropriate set of axioms in your language: (2) Groups. *[3]*

5.11. *[Problem 5.11]* Give a precise definition of the scope of a quantifier. *[2]*

5.12. *[Problem 5.12]* Which of the formulas you gave in solving Problem 5.8 are sentences? *[1 = 2×0.5 each]*

[Total = 15]