Mathematics-Computer Science 4215H – Mathematical Logic
TRENT UNIVERSITY, Winter 2021

Assignment #4
Due on Friday, 12 February.

Do all of the following problems, which are straight out of the textbook[0] (which explains the numbering), reproduced here for your convenience.

3.9. *[Problem 3.9]* Appealing to previous deductions and the Deduction Theorem if you wish, show that:
(1) $\{\delta, \neg\delta\} \vdash \gamma$ *[1]*
(2) $\vdash \varphi \to \neg\neg\varphi$ *[4]*

NOTE. You may assume any and all the examples, problems, and results of Chapter 3, up to 3.8 inclusive, when doing **3.9** (1) & (2).

4.3. *[Proposition 4.3]* Suppose Δ is an inconsistent set of formulas. Then $\Delta \vdash \psi$ for any formula ψ. *[2]*

4.4. *[Proposition 4.4]* Suppose Σ is an inconsistent set of formulas. Then there is a finite subset Δ of Σ such that Δ is inconsistent. *[2]*

4.5. *[Corollary 4.5]* A set of formulas Γ is consistent if and only if every finite subset of Γ is consistent. *[2]*

4.8. *[Proposition 4.8]* Suppose Σ is a maximally consistent set of formulas and φ is a formula. Then $\neg\varphi \in \Sigma$ if and only if $\varphi \notin \Sigma$. *[2]*

4.9. *[Proposition 4.9]* Suppose Σ is a maximally consistent set of formulas and φ and ψ are formulas. Then $\varphi \to \psi \in \Sigma$ if and only if $\varphi \notin \Sigma$ or $\psi \in \Sigma$. *[2]*

[Total = 15]

[0] *A Problem Course in Mathematical Logic*, Version 1.6.