Mathematics-Computer Science 4215H – Mathematical Logic

TRENT UNIVERSITY, Winter 2021

Assignment #3

Due on Friday, 5 February.

Do all of the following problems, which are straight out of the textbook[0] (which explains the numbering), reproduced here for your convenience.

3.1. *[Proposition 3.1]* Every axiom of $\mathcal{L}_P$ is a tautology. *[3]*

3.2. *[Proposition 3.2]* Suppose φ and ψ are formulas. Then $\{\, \varphi, (\varphi \to \psi) \,\} \vdash \psi$. *[1]*

3.4. *[Proposition 3.4]* If $\varphi_1\varphi_2 \ldots \varphi_n$ is a deduction of $\mathcal{L}_P$, then $\varphi_1 \ldots \varphi_\ell$ is also a deduction of $\mathcal{L}_P$ for any ℓ such that $1 \leq \ell \leq n$. *[1]*

3.7. *[Proposition 3.7]* If $\Gamma \vdash \Delta$ and $\Delta \vdash \sigma$, then $\Gamma \vdash \sigma$. *[3]*

3.8. *[Theorem 3.8 – Deduction Theorem]* If Σ is any set of formulas and α and β are any formulas, then $\Sigma \vdash \alpha \to \beta$ if and only if $\Sigma \cup \{\alpha\} \vdash \beta$. *[5]*

3.9(3). *[Proposition 3.9(3)]* Appealing to previous deductions and the Deduction Theorem if you wish, show that $\vdash (\neg\beta \to \neg\alpha) \to (\alpha \to \beta)$. *[2]*

NOTE. You may assume any and all the examples, problems, and results of Chapter 3, up to **3.9**(2) inclusive, when doing **3.9**(3).

[Total = 15]

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[0] *A Problem Course in Mathematical Logic*, Version 1.6.