

## Mathematics-Computer Science 4215H – Mathematical Logic

TRENT UNIVERSITY, Winter 2021

### Assignment #2

*Due on Friday, 29 January.*

Do all of the following problems, which are straight out of Chapter 1 of the textbook<sup>0</sup> (which explains the numbering), reproduced here for your convenience.

**2.1.** [Problem 2.1] Suppose  $v$  is a truth assignment such that  $v(A_0) = v(A_2) = T$  and  $v(A_1) = v(A_3) = F$ . Find  $v(\alpha)$  if  $\alpha$  is:

(1)  $\neg A_2 \rightarrow \neg A_3$

(3)  $\neg(\neg A_0 \rightarrow A_1)$

(5)  $A_0 \wedge A_1$

[1.5 = 3 × 0.5 each]

**2.2.** [Proposition 2.2] Suppose  $\delta$  is any formula and  $u$  and  $v$  are truth assignments such that  $u(A_n) = v(A_n)$  for all atomic formulas  $A_n$  which occur in  $\delta$ . Show that  $u(\delta) = v(\delta)$ . [4.5]

**2.3.** [Corollary 2.3] Suppose  $u$  and  $v$  are truth assignments such that  $u(A_n) = v(A_n)$  for every atomic formula  $A_n$ . Show that  $u = v$ , i.e.  $u(\varphi) = v(\varphi)$  for every formula  $\varphi$ . [1]

**2.7.** [Proposition 2.7] Show that if  $\Gamma$  and  $\Sigma$  are sets of formulas such that  $\Gamma \subseteq \Sigma$ , then  $\Sigma \models \Gamma$ . [1]

**2.8.** [Problem 2.8] How can one check whether or not  $\Sigma \models \varphi$  for a formula  $\varphi$  and a finite set of formulas  $\Sigma$ ? [3]

**2.10.** [Proposition 2.10] Show that a set of formulas  $\Sigma$  is satisfiable if and only if there is no contradiction  $\chi$  such that  $\Sigma \models \chi$ . [4]

[Total = 15]

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<sup>0</sup> A Problem Course in Mathematical Logic, Version 1.6.