Mathematics-Computer Science 4215H – Mathematical Logic

TRENT UNIVERSITY, Winter 2021

Assignment #11

Due on Friday, 9 April.

Do all of the following problems, all of which are straight out of the textbook[0] (which explains the numbering), reproduced here for your convenience.

8.11. *[Proposition 8.11]* Suppose Γ and Σ are sets of sentences of $\mathcal{L}$, $\Gamma \subseteq \Sigma$, and C is a set of witnesses for Γ in $\mathcal{L}$. Then C is a set of witnesses for Σ in $\mathcal{L}$. *[2]*

8.12. *[Lemma 8.12]* Suppose Σ is a set of sentences, φ is any formula, and x is any variable. Then $\Sigma \vdash \varphi$ if and only if $\Sigma \vdash \forall x\, \varphi$. *[3]*

8.13. *[Theorem 8.13]* Suppose Γ is a consistent set of sentences of $\mathcal{L}$. Let C be an infinite countable set of constant symbols which are *not* symbols of $\mathcal{L}$, and let $\mathcal{L}' = \mathcal{L} \cup C$ be the language obtained by adding the constant symbols in C to the symbols of $\mathcal{L}$. Then there is a maximally consistent set Σ of sentences of $\mathcal{L}'$ such that $\Gamma \subseteq \Sigma$ and C is a set of witnesses for Σ. *[5]*

8.16. *[Theorem 8.16]* A set of sentences Σ in $\mathcal{L}$ is consistent if and only if it is satisfiable. *[3]*

8.17 *[Theorem 8.17] (Completeness Theorem)* If α is a sentence and Δ is a set of sentences such that $\Delta \vDash \alpha$, then $\Delta \vdash \alpha$. *[2]*

[Total = 15]

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[0] *A Problem Course in Mathematical Logic*, Version 1.6.