Mathematics-Computer Science 4215H – Mathematical Logic
TRENT UNIVERSITY, Winter 2021
Assignment #10
Due on Monday, 5 April.

Do all of the following problems, all of which are straight out of the textbook[0] (which explains the numbering), reproduced here for your convenience.

7.3. *[Problem 7.3]* Determine whether or not each of the following formulas is a logical axiom.

(1) $\forall x \, \forall z \, (x = y \rightarrow (x = c \rightarrow x = y))$ *[0.5]*
(3) $\forall z \, (x = y \rightarrow (x = c \rightarrow y = c))$ *[0.5]*
(5) $\forall x \, (\forall x \, c = fxc \rightarrow \forall x \, \forall x \, c = fxc)$ *[0.5]*

7.5. *[Problem 7.5]* Show that:

(1) $\vdash \forall x \, \varphi \rightarrow \forall y \, \varphi_y^x$, if y does not occur at all in φ. *[1.5]*
(3) $\{c = d\} \vdash \forall z \, Qazc \rightarrow Qazd$. *[1.5]*
(5) $\{\exists x \, \alpha\} \vdash \alpha$ if x does not occur free in α. *[1.5]*

7.13. *[Problem 7.13]* Show that:

(1) $\vdash \forall x \, \forall y \, \forall z \, (x = y \rightarrow (y = z \rightarrow x = z))$. *[2]*
(2) $\vdash \forall x \, \alpha \rightarrow \exists x \, \alpha$. *[2]*

8.1. *[Theorem 8.1]* *(Soundness Theorem)* If α is a sentence and Δ is a set of sentences such that $\Delta \vdash \alpha$, then $\Delta \vDash \alpha$. *[3]*

8.2 *[Proposition 8.2]* If a set of sentences Γ is satisfiable, then it is consistent. *[2]*

[Total = 15]