

## Mathematics-Computer Science 4215H – Mathematical Logic

TRENT UNIVERSITY, Winter 2021

### Assignment #1

*Due on Friday, 22 January.*

Do all of the following problems, which are straight out of Chapter 1 of the text-book<sup>0</sup> (which explains the numbering), reproduced here for your convenience with a few explanations in the added footnotes.

- 1.1.** [Problem 1.1] Why are the following *not* formulas of  $\mathcal{L}_P$ ? There might be more than one reason ... [1.5 = 3 × 1.5 each]
- (1)  $A_{-56}$
  - (3)  $A_7 \leftarrow A_4$
  - (5)  $(A_8 A_9 \rightarrow A_{1043998})$
- 1.5.** [Problem 1.5] What are the possible lengths<sup>1</sup> of formulas of  $\mathcal{L}_P$ ? Prove it. [5]
- 1.7.** [Proposition 1.7] Show that the set of formulas of  $\mathcal{L}_P$  is countable<sup>2</sup>. [5]
- 1.9.** [Problem 1.9] Write out  $((\alpha \vee \beta) \wedge (\beta \rightarrow \alpha))$  using only  $\neg$  and  $\rightarrow$ . [1.5]
- 1.11.** [Problem 1.11] Find all the subformulas of each of the following formulas<sup>3</sup>. [2 = 2 × 1 each]
- (1)  $(\neg((\neg A_{56}) \rightarrow A_{56}))$
  - (2)  $A_9 \rightarrow A_8 \rightarrow \neg(A_{78} \rightarrow \neg\neg A_0)$

[Total = 15]

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<sup>0</sup> A Problem Course in Mathematical Logic, Version 1.6.

<sup>1</sup> As sequences of symbols.

<sup>2</sup> That is, all the elements of the set can be put into a list indexed by the natural numbers.

<sup>3</sup> That is, of their official versions.