MATH 4215H — Deductions in first-order logic II - A bit more on substitution

Official defns last time (or in the textbook), but informally t is substitutable for x in φ if no variable in t gets captured by a quantifier if t is used to replace x in every <u>free</u> occurrence of x in φ. If it is, then φ^x_t is the formula in which every free occurrence of x has been replaced by t.

term · variable · formula

Example: φ is $\forall y \forall x \overbrace{(\underbrace{Pxfy}_{\alpha} \to \forall y\, Pxfy)}^{\psi}$,

where P is a 2-place relation symbol and f is a 1-place function symbol.

(i) If t is fa for some constant symbols a ~~&~~ b, then t is substitutable for x in φ, but φ^x_t is just φ since no instance of x in φ is free.

(2) If t is fa for constant symbols a then t is substitutable for y in $\varphi \equiv \forall x(Pxf\underset{\text{free}}{y} \to \forall y Pxf\underset{\text{not free}}{y})$ and φ^y_t is

bound to

$$\forall x(Pxffa \to \forall_y Pxfy).$$

(3) If t is fy then t is substitutable for y in φ and φ^y_t $\forall x(Pxffy \to \forall_y Pxfy)$

(4) If t is fx then t is not substitutable for y in φ since $\forall x(Pxffx \to \forall_y Pxfy)$.

captured by

(5) If t is fx then t is substitutable for y in α

$$\text{is } \quad Pxfy \to \forall_y Pxfy$$

and α^y_t is $Pxffx \to \forall_y Pxfy$.

<u>Note:</u> Most facts about deductions & and how they work are the same as for propositional logic. Look at differences next time!