MATH 4215H First-Order Logic, Overview

Propositional logic captures what connectives like "or", "and", "not", "if...then...", etc can do. First-order logic adds what quantifiers "for all" and "there exists" can do. This lets us do essentially all of math, but has a much higher cost in terms of infrastructure in defining the language(s), defining how truth values work, and ditto for deductions.

First-order languages normally include

(1) connectives: $\neg$, $\rightarrow$ (& informally, $\vee$, $\wedge$, $\leftrightarrow$, ...)

(2) parentheses: (,)

(3) quantifier: $\forall$ (& informally, $\exists$)

(4) variables: $x_0, x_1, x_2, \ldots$ [& informally, $x, y, z, \ldots$]

(5) equality: $=$

They *may* also include:

(6) constants: $c_0, c_1, c_2, \ldots$ [& informally $a, b, c, \ldots$ & common symbols like $0, 1, \ldots$]

(7) functions: $f_0, f_1, f_2, f_3, \ldots$ [& informally $f, g, h, \ldots$ & common symbols like $+, -, \cdot, \div$, etc. if appropriate]

(8) relations: $P_0, P_1, P_2, \ldots$ [& informally, $P, Q, R, \ldots$ & common symbols like $<, >, \leq, \geq, |, \|, \perp$, ∘∘∘]
(predicates)

... depending on the application one has in mind.

The idea is that one has an "universe of discourse" [a mathematical structure like a group, or a number system, or a vector space, or a geometry, ...] with objects that are named by constants & varied over by the variable, operated on by functions, *etc*

This is an informal start... The details will come all too soon — so start reading Ch. 5.