MATH-COIS 4215H - Truth & Consequences

2021-01-18 ①

(Chapter 2 in the textbook.)

We'll be using a classical "two-valued" logic with the "truth value", T (true) and F (false). Recall from 2200H, or elsewhere, that we can use truth tables to define how the connectives interact with truth values.

A	$\neg A$
T	F
F	T

antecedent consequent

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

We'll need a more formal and precise approach (still be compatible with truth tables as we know them).

Def'n: A truth assignment decides the truth value of each atomic formula. Formally, a truth assignment is a function v whose domain is the set of all atomic formulas of $\mathcal{L}_p$ and whose range is the set $\{T, F\}$ of truth values.

ie $v(A_n)$ gives us T or F (for each atomic formula A_n)

In terms of truth tables:

A_0	A_1	A_2	...
⋮			
$v(A_0)$	$v(A_1)$	$v(A_2)$	...

- each truth assignment is like a row of the ultimate truth table for $\mathcal{L}_p$. (at least for atomic formulas)

Def'n: (cont'd) We extend each truth assigment on the atomic formulas to all the formulas of $\mathcal{L}_P$ as follows:

(1) $v(A_n)$ is defined for all A_n

(2) For any formula α for which $v(\alpha)$ has been defined, let $v(\neg\alpha) = \begin{cases} T & \text{if } v(\alpha) = F \\ F & \text{if } v(\alpha) = T \end{cases}$.

(3) For any formulas α and β for which $v(\alpha)$ and $v(\beta)$ are defined,

let $v(\alpha \to \beta) = \begin{cases} F & \text{if } v(\alpha) = T \text{ and } v(\beta) = F \\ T & \text{otherwise} \end{cases}$.

$\Rightarrow$ Suppose we have the formula $A_0 \to (\neg(A_1 \to A_2))$ and our truth assignment v has the values $v(A_0) = T$, $v(A_1) = F$, $v(A_2) = T$ for the atomic formulas occurring in the given formula.

Then (hard way)

$$v(A_0 \to (\neg(A_1 \to A_2)))$$

$$= \begin{cases} F & \text{if } v(A_0) = T \;\&\; v(\neg(A_1 \to A_2)) = F \\ T & \text{otherwise} \end{cases}$$

But $v(A_0) = T$, so we need to know $v(\neg(A_1 \to A_2))$.

$$v(\neg(A_1 \to A_2)) = \begin{cases} T & \text{if } v(A_1 \to A_2) = F \\ F & \text{if } v(A_1 \to A_2) = T \end{cases}$$

Since $v(A_1 \to A_2) = \begin{cases} F & \text{if } v(A_1) = T \;\&\; v(A_2) = F \\ T & \text{otherwise} \end{cases}$

and $v(A_1) = F$ & $v(A_2) = T$, $v(A_1 \to A_2) = T$,
and thus $v(\neg(A_1 \to A_2)) = F$, and hence
$v(A_0 \to (\neg(A_1 \to A_2))) = F$.

The easy way here would be to use a truth table:

	A_0	A_1	A_2	$A_1 \to A_2$	$\neg(A_1 \to A_2)$	$A_0 \to (\neg(A_1 \to A_2))$
$v(\,)$	T	F	T	T	F	F

✓

Next time: We'll look at how collections of formulas imply formulas in terms of truth.