## Mathematics-Computing & Information Systems 4215H – Mathematical logic TRENT UNIVERSITY, Fall 2012

## **Take-home Final Examination**

Due on Wednesday, 19 December, 2012.

**Instructions:** All references below are to the text. You may consult whatever sources you wish, so long as you acknowledge those you actually use. However, you may not consult other people, except to ask the instructor to clarify problems.

**Part I.** Do any *two* (2) of 1–4.  $/20 = 2 \times 10$  *each*/

- **1.** Problem 1.3
- 2. Proposition 2.9
- **3.** Problem 3.9 (8)
- **4.** Theorem 4.7

**Part II.** Do any three (3) of 5-9.  $[30 = 3 \times 10 \text{ each}]$ 

- **5.** Theorems 5.15 and 5.16
- **6.** Proposition 6.16 parts (1) and (2)
- 7. Proposition 6.17
- 8. Proposition 7.4 (for axiom schema (A4) only)
- **9.** Theorems 8.17 and 8.18

**Part III.** Do any one (1) of **10–12**. [15]

10. Prove the Interpolation Theorem for propositional logic:

Suppose  $\alpha$  and  $\beta$  are formulas of  $\mathcal{L}_P$  such that  $\vdash \alpha \rightarrow \beta$ . Show that there is a formula  $\gamma$  of  $\mathcal{L}_P$  such that both  $\vdash \alpha \rightarrow \gamma$  and  $\vdash \gamma \rightarrow \beta$ , and every atomic formula that appears in  $\gamma$  either appears in both or neither of  $\alpha$  and  $\beta$ .

- **11.** Theorem 7.12
- 12. Proposition 9.4

[Total = 65]

## Part IV. Bonus!

{}. Write an original limerick about logic. (There are some examples in Appendix C.) [2]

I HOPE YOU ENJOYED THE COURSE. HAVE A GOOD HOLIDAY!