

**Mathematics 3820H – Mathematics from medieval to modern times**

TRENT UNIVERSITY, Winter 2019

TAKE-HOME FINAL EXAMINATION

*Due on Friday, 12 April, 2019.*

**Instructions:** Give complete answers to receive full credit, including references to any and all sources you used. You may ask the instructor to clarify the instructions or any of the questions, use a calculator or computer to perform any necessary calculations, and consult any sources you wish, *with the exception of other students' work*, and you may not give or receive any other aid on this exam, except with the instructor's explicit permission.

**Part I – Various things, ...** Do all three (3) of 1 – 3.

1. Answer all of a – j. [10 = 10 × 1 each]
  - a. Who established the complex numbers as a number system in its own right?
  - b. Who first used the symbol = to denote equality?
  - c. What are the origins of the word “algebra”?
  - d. Name a mathematician who was also (in)famous as an engineer, doctor, and gambler.
  - e. Name three priority disputes involving work in mathematics.
  - f. Who showed that not having parallel lines is inconsistent with Postulates I–IV of the *Elements*?
  - g. Name two mathematicians who also wrote poetry.
  - h. Who named geometric objects and concepts after parts of flowers?
  - i. Name an early critic of the lack of rigour in the foundations of calculus.
  - j. Name a mathematician who was reputed to be a sorcerer.
2. A lot of the mathematicians we talked about were also astronomers, at least in part. Assess, as best you briefly can, to what extent the development of mathematics was driven by the needs of astronomy. [15]
3. Describe the method given by Euler in Part I, Section II, Chapter IX (starting on p. 155 in the translation noted below) of his *Elements of Algebra* (2nd Edition), Vol. I, for computing cube roots in modern terms, and use it to find the cube root of 19683. [10]

NOTE: There is an 1810 translation online at: [books.google.ca/books?id=hqI-AAAAYAAJ](http://books.google.ca/books?id=hqI-AAAAYAAJ)

[Parts II-IV are on page 2.]

**Part II – ... and older things, ...** Do *one* (1) of **4** or **5**.

4. After calculus was introduced by Newton and Leibniz, mathematicians and scientists used and continued to develop it for nearly two centuries before it was given a more or less rigorous foundation. Explain why they did so and whether they were justified in doing so. [15]
5. Trace the spread and evolution of the Hindu number system and try to assess to what extent it spread because of its superiority to the available alternatives. [15]

NOTE: *The Hindu-Arabic Numerals*, by L.C. Karpinski and D.E. Smith, may be useful. It can be found at Project Gutenberg: [www.gutenberg.org/ebooks/22599](http://www.gutenberg.org/ebooks/22599)

**Part III – ... and ~~poofy~~ proofy things, ...** Do any *two* (2) of **6** – **8**.

6. State and prove Brahmagupta's formula for the area of a cyclic quadrilateral. [10]
7. Prove *Fermat's Little Theorem*: If  $p$  is a prime and  $a > 0$  is an integer which is not a multiple of  $p$ , then  $a^{p-1} - 1$  is a multiple of  $p$ . [10]
8. State and prove Newton's Generalized Binomial Theorem. [10]

[Total = 70]

**Part IV - ... and even verse things.** Bonus!

- 3<sup>2</sup>. Write an original poem touching on mathematics or its history. [1]
- $\sqrt{100}$ . Find and give a complete reference to a poem touching on mathematics or its history that you did not write, and which your instructor has not seen before. [1]

I HOPE THAT YOU ENJOYED THE COURSE,  
AND HAVE AN EVEN BETTER TIME THIS SUMMER!