Mathematics 3820H – Mathematics from medieval to modern times
TRENT UNIVERSITY, Winter 2018

Assignment #2
Extracting Understanding
Due on Friday, 1 February, 2019.

The Persian polymath Omar al-Khayyami (*c.* 1050–1123 A.D.), is best known nowadays as a poet [1]. He did considerable work in astronomy and mathematics, developing geometric techniques for finding the positive real roots of cubic and quartic equations*. His method for solving certain cubic equations can be described – much more succinctly than he was able to – as follows in modern terms:

> To solve the cubic equation $x^3 + ax^2 + b^2x + b^2c = 0$, intersect the hyperbola $y = \dfrac{bc}{x} + b$ with the circle $\left(x + \frac{1}{2}(a+c)\right)^2 + y^2 = \frac{1}{4}(a-c)^2$ and find the point of intersection other than $(-c, 0)$.

1. Verify that al-Khayyami's method for solving cubic equations actually does find positive real roots, if such exist. *[5]*

I couldn't find a freely available translation of al-Khayyami's work on algebra into English, but there is one into French [2] if anyone wishes to look it up.

2. Give a geometrical method – in modern terms, please – for solving (a non-trivial class of) quartic equations and verify that it works. *[5]*

REFERENCE

1. *Rubaiyat of Omar Khayyam*, trans. by Edward Fitzgerald, London, 1859 [1st edition]. The text may be found online at: `http://www.omarkhayyamrubaiyat.com/text.htm`
2. *L'algèbre d'Omar Alkhayyammi*, trans. by F. Woepcke, Paris, 1851. (Arabic with French Translation.) Available online (pdf) at:
   `http://www.wilbourhall.org/pdfs/algebraOmarAlkhayyami.pdf`

* He apparently tried and failed to find purely algebraic methods, but expressed the hope that later mathematicians would succeed in doing so. Tartaglia and Cardano eventually realized this hope in the 1500s.